Discrete Reconstruction for Electron Tomography

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28 Août 2015

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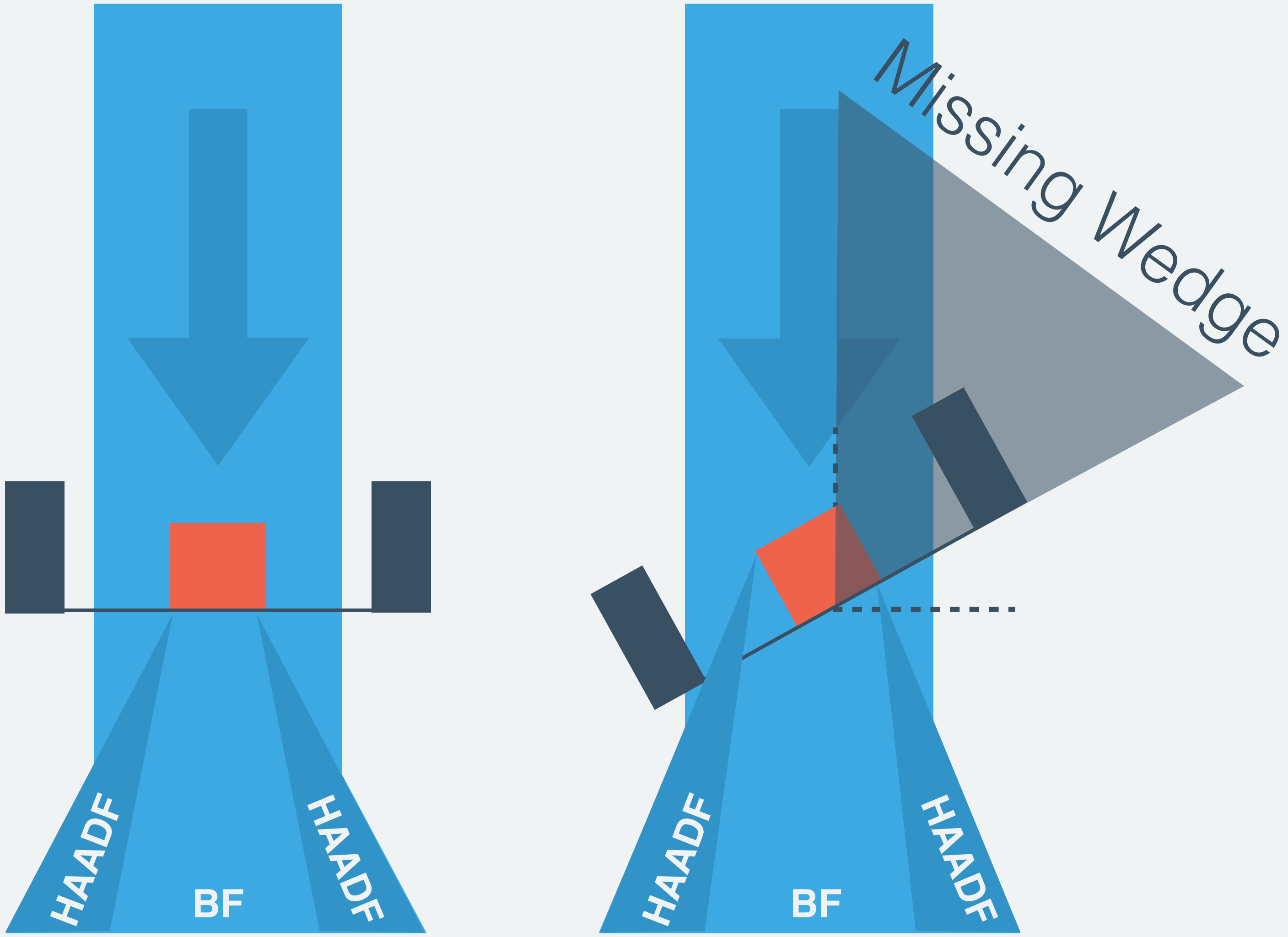
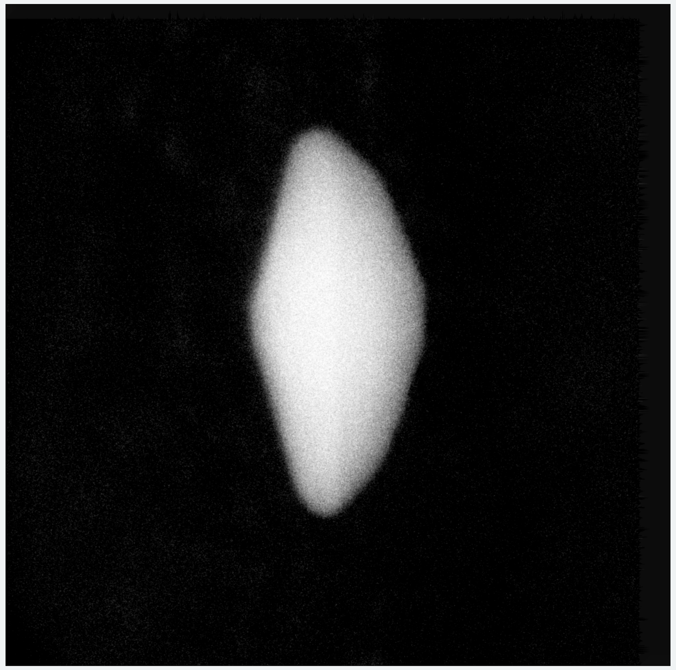
ERC

STEM for Tomography

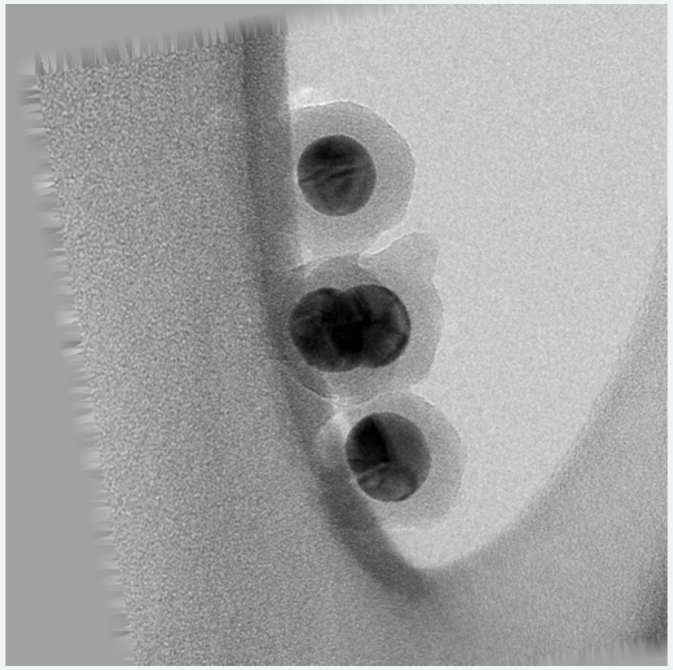
Acquisition

Series of micrographs acquired at varying sample tilt angles.





HAADF Mode

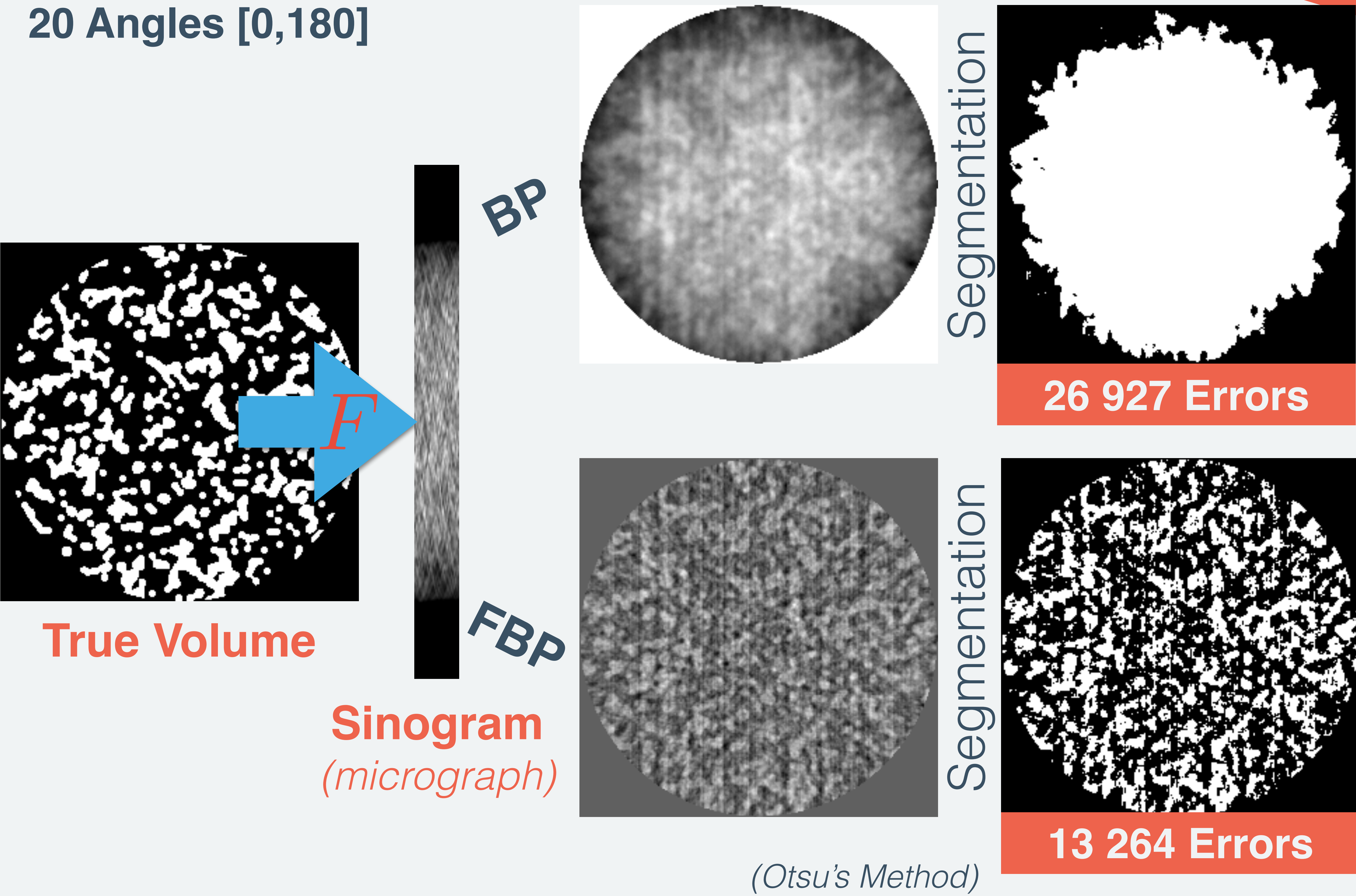


Bright Field

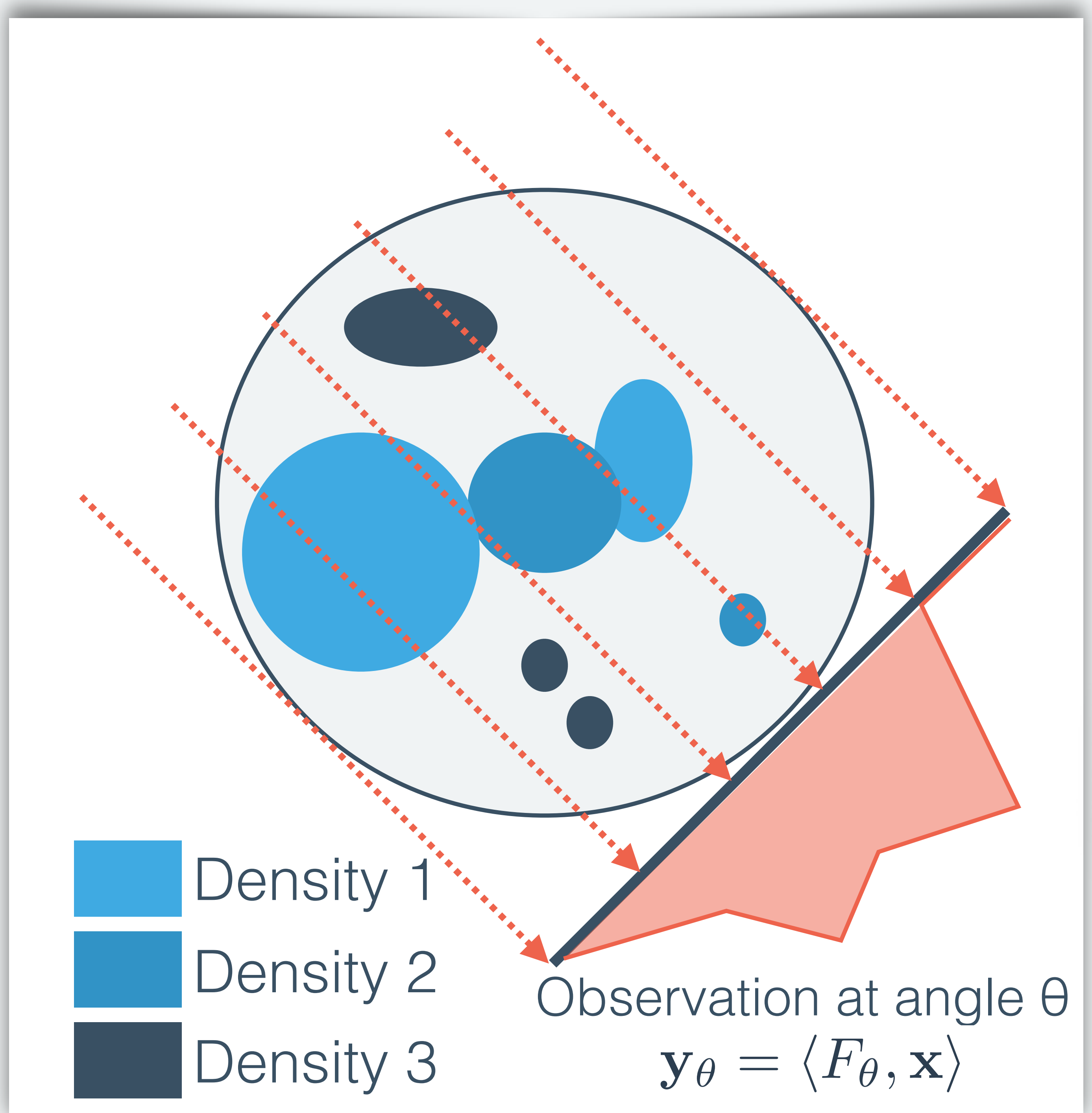
Goal: Recover volume/model of specimen from minimal number of tilt-angle micrographs.

One-step Reconstruction (2D)

20 Angles [0,180]



Tomography as Linear Problem

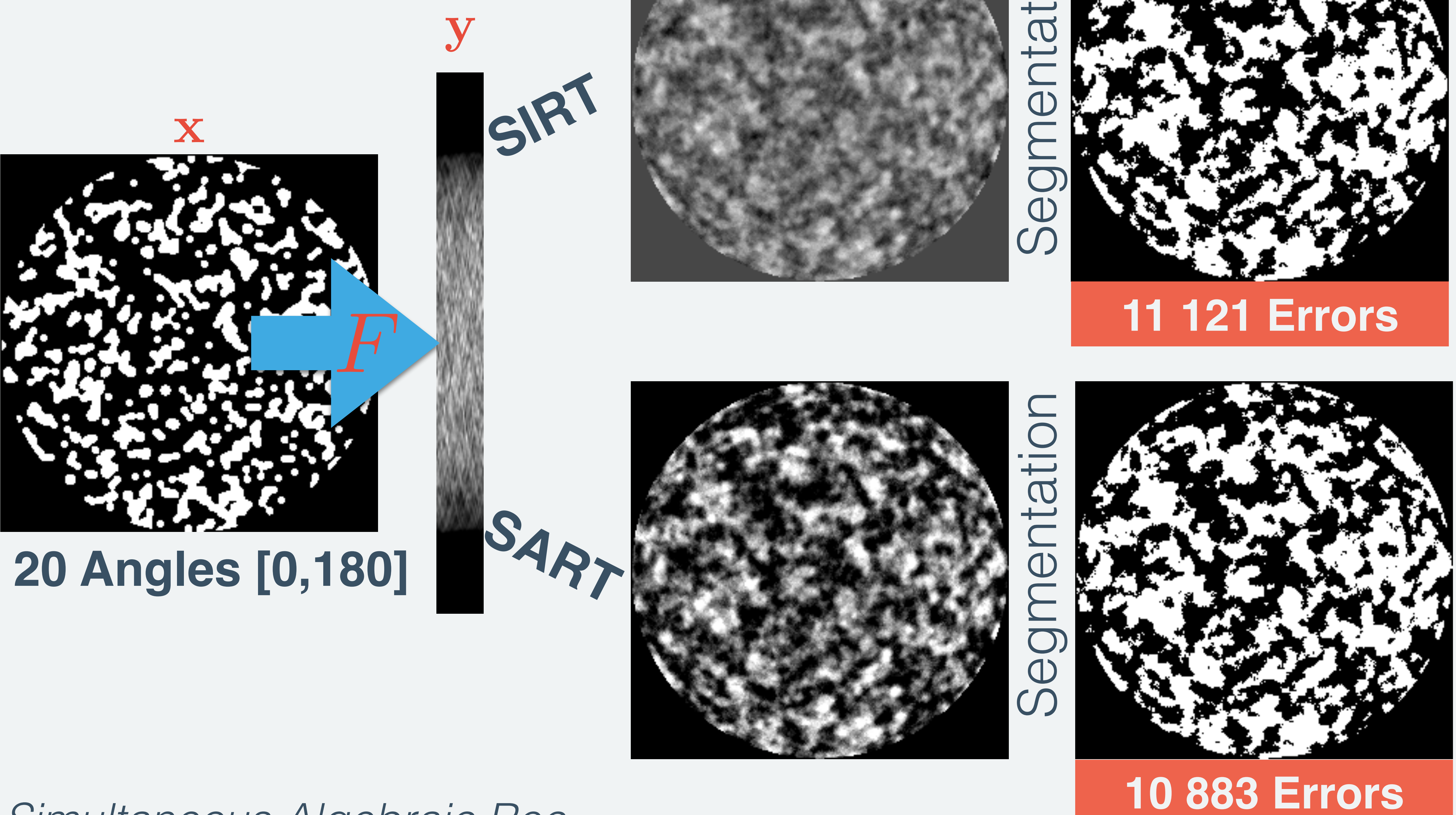


Tomographic Recovery is essentially solving a linear system of equations.

$$\mathbf{y} = F\mathbf{x} + \mathbf{w}$$ *possible noise*

Algebraic Recovery Methods (ARM)

Simultaneous Iterative Rec.

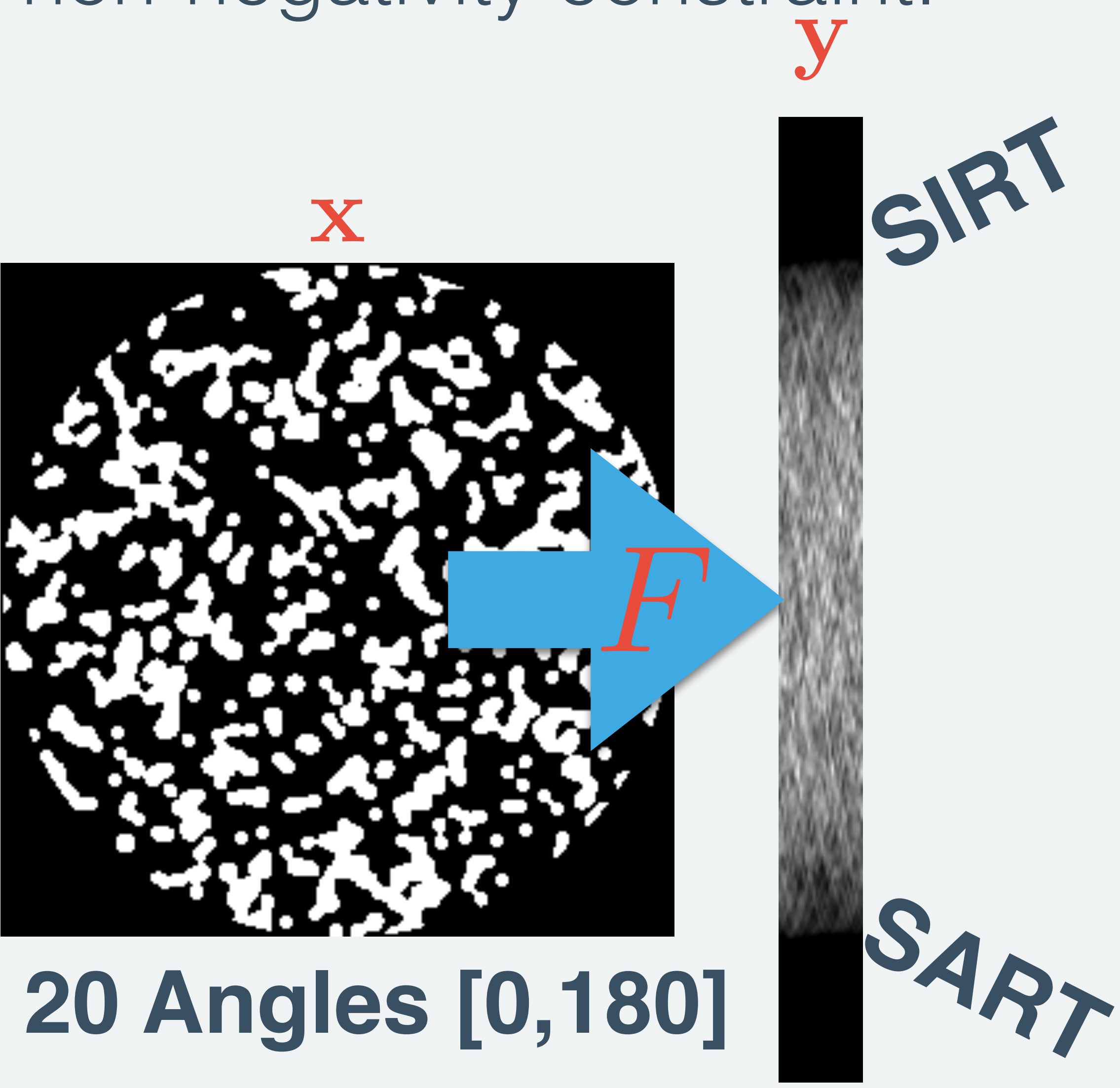


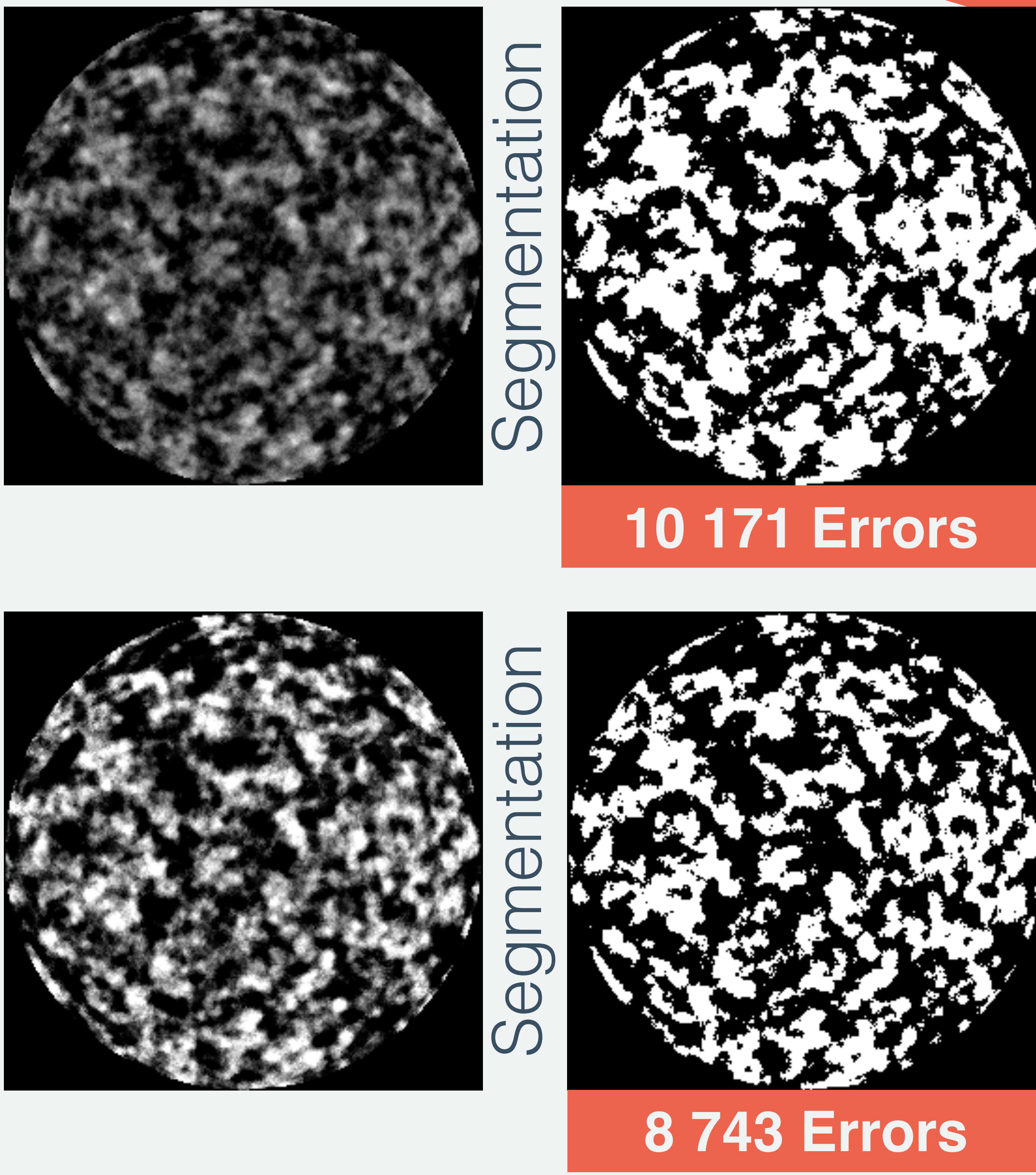
Simultaneous Algebraic Rec.

Algebraic Recovery Methods (ARM)

Prior Information

Iterative ARM allows for non-negativity constraint.



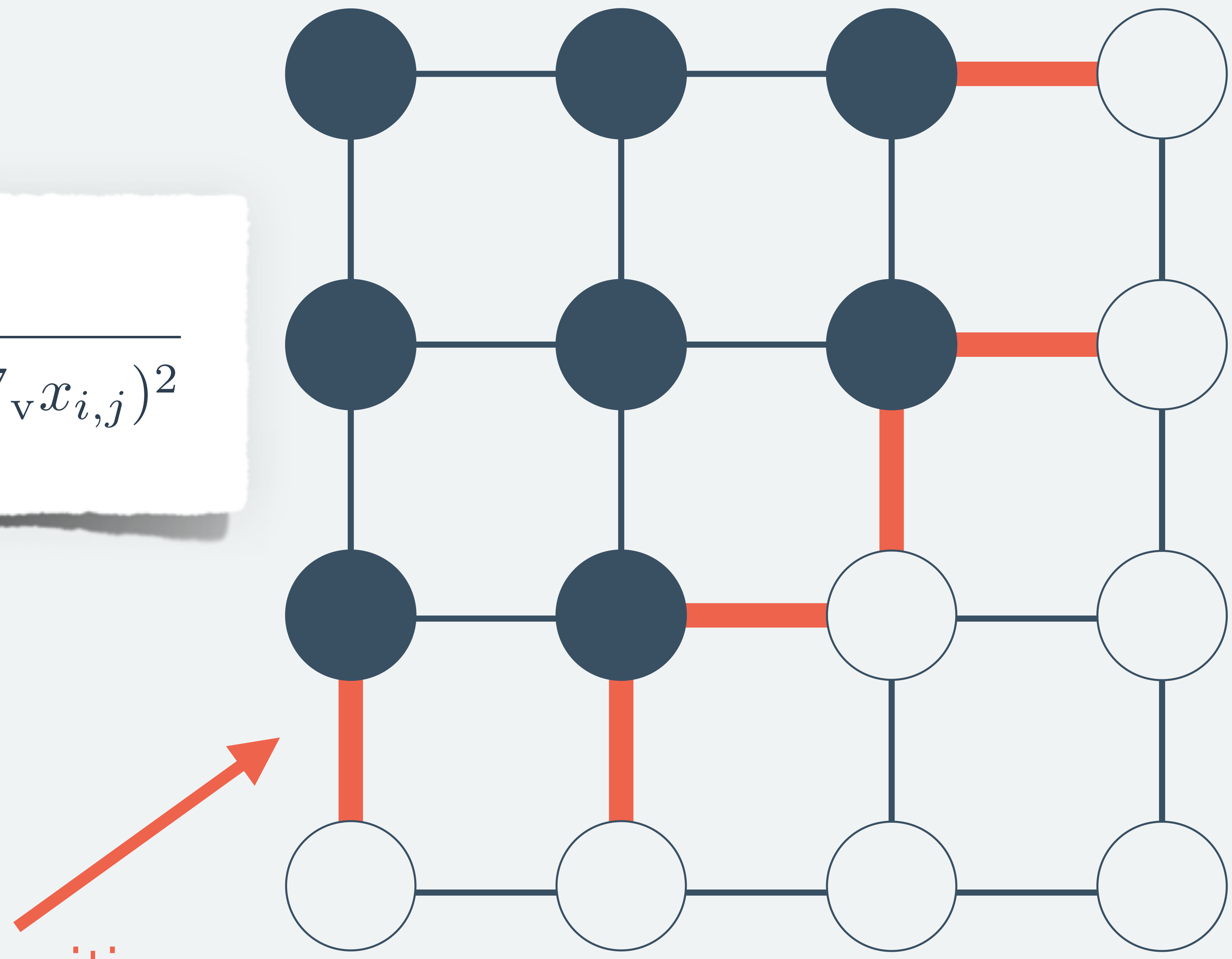


Total Variation Minimization

Prior Information:

Reconstructed images should be "smooth".

$$\min_{\mathbf{x}} \quad ||\mathbf{x} - F\mathbf{y}||_2^2 + \sum_{i,j} \sqrt{(\nabla_{\mathrm{h}} x_{i,j})^2 + (\nabla_{\mathrm{v}} x_{i,j})^2}$$

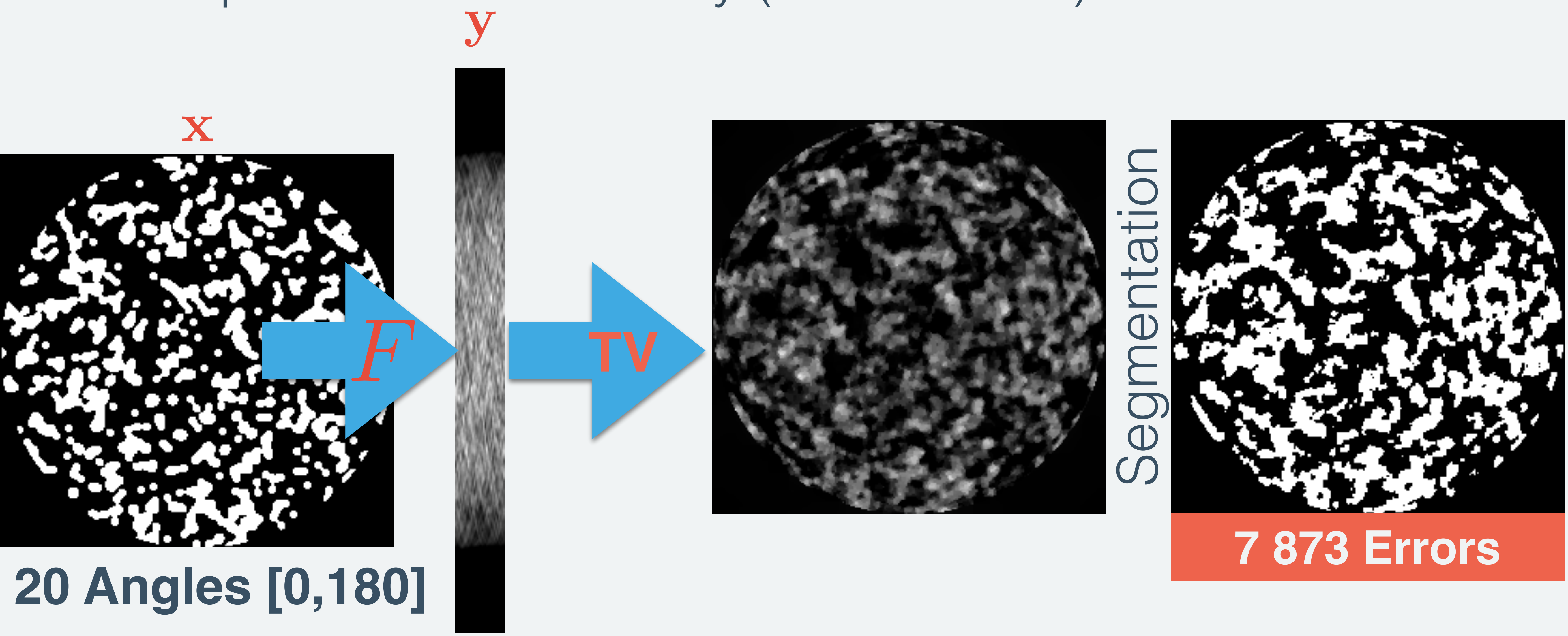


e.g. penalize discontinuities in image.

Total Variation Minimization

Prior Information

Enforce piece-wise continuity (smoothness)



20 Angles [0,180]

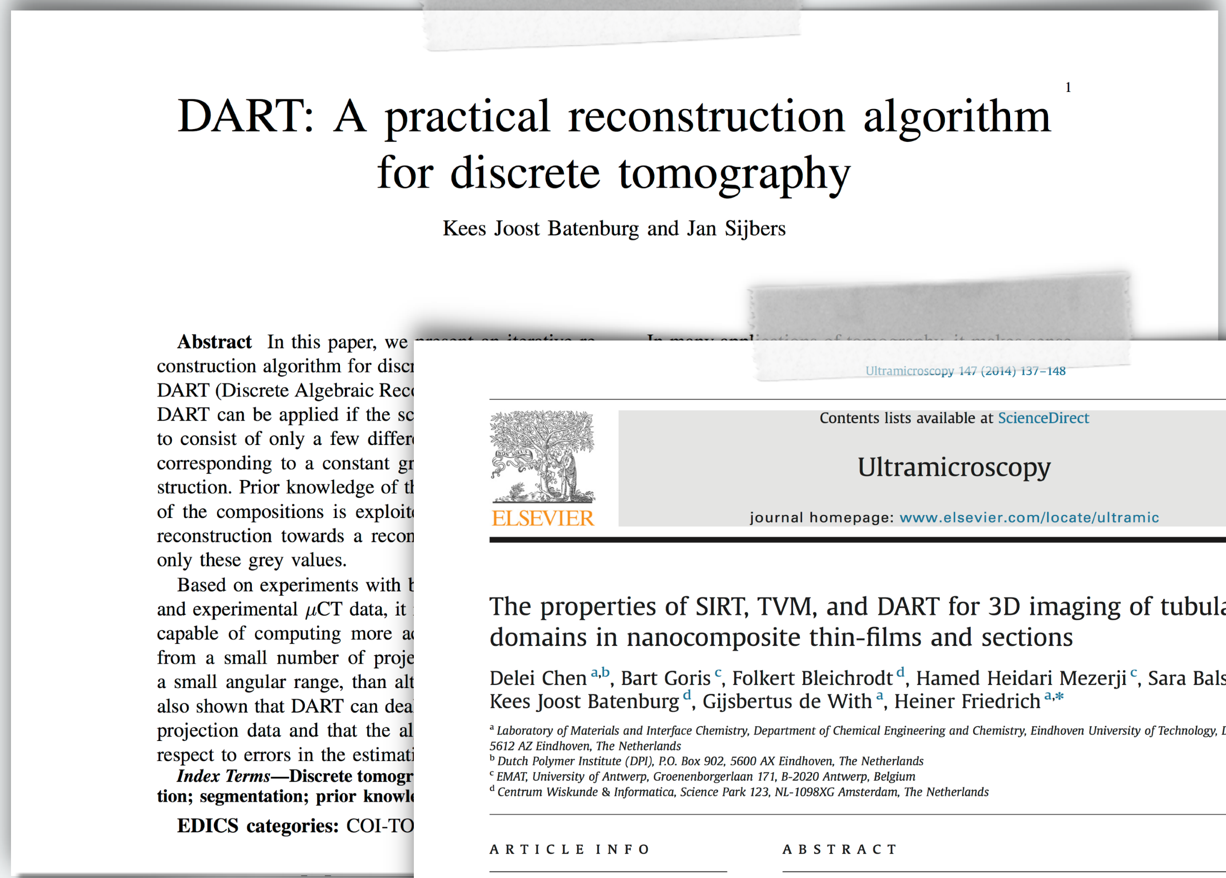
Discrete ART (DART)

Prior Information:

Elements (pixels/voxels) of the volume belong to a small set of values.

➡ An EM-like procedure on top of ARM reconstruction.

➡ Inherently greedy/empirical technique



DART: A practical reconstruction algorithm for discrete tomography

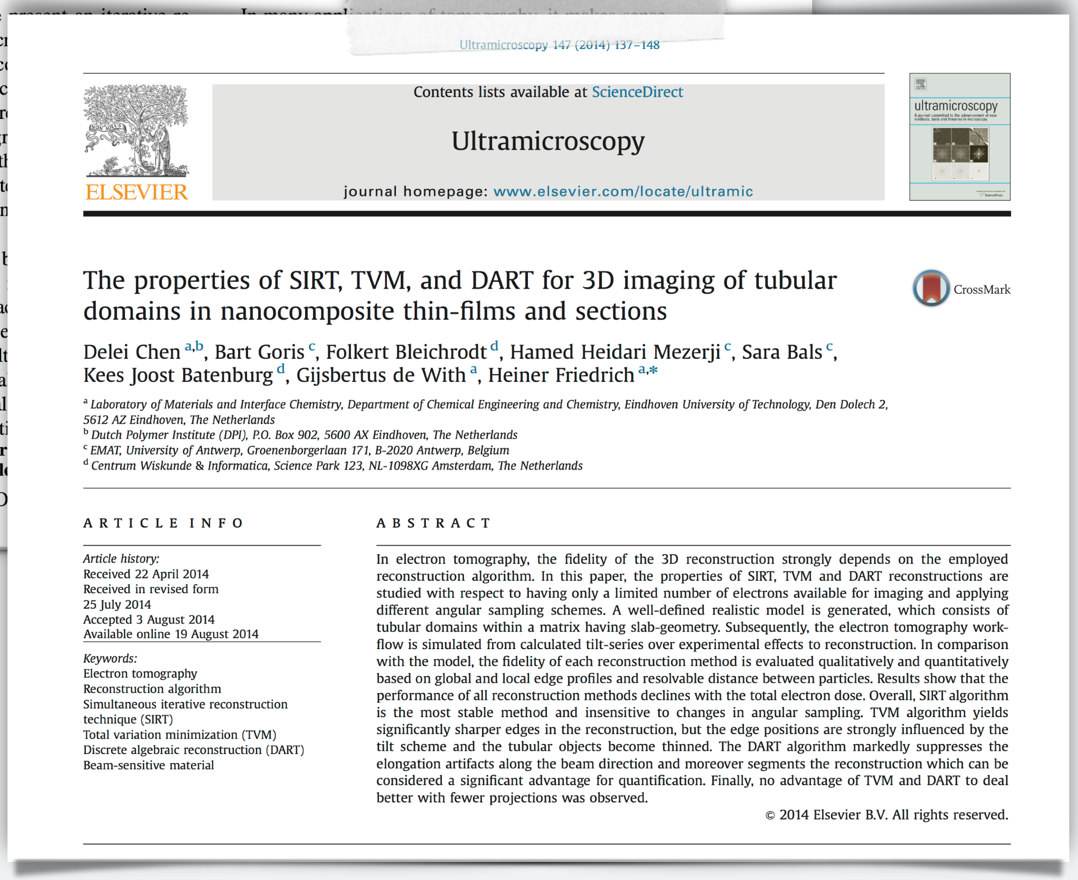
Kees Joost Batenburg and Jan Sijbers

Abstract In this paper, we
construction algorithm for discr
DART (Discrete Algebraic Reco
DART can be applied if the sc
to consist of only a few differ
corresponding to a constant gr
struction. Prior knowledge of th
of the compositions is exploite
reconstruction towards a recon
only these grey values.

Based on experiments with b
and experimental μCT data, it
capable of computing more ac
from a small number of proje
a small angular range, than alt
also shown that DART can dea
projection data and that the al
respect to errors in the estimati

Index Terms—Discrete tomogr
tion; segmentation; prior knowle

EDICS categories: COI-TO



Ultramicroscopy 147 (2014) 137–148

Contents lists available at ScienceDirect

Ultramicroscopy

journal homepage: www.elsevier.com/locate/ultramic

The properties of SIRT, TVM, and DART for 3D imaging of tubular domains in nanocomposite thin-films and sections

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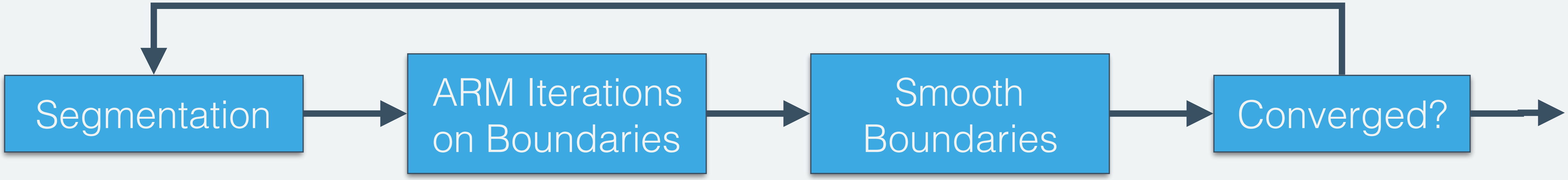
ARTICLE INFO

Article history:
Received 22 April 2014
Received in revised form
25 July 2014
Accepted 3 August 2014
Available online 19 August 2014

Keywords:
Electron tomography
Reconstruction algorithm
Simultaneous iterative reconstruction technique (SIRT)
Total variation minimization (TVM)
Discrete algebraic reconstruction (DART)
Beam-sensitive material

ABSTRACT

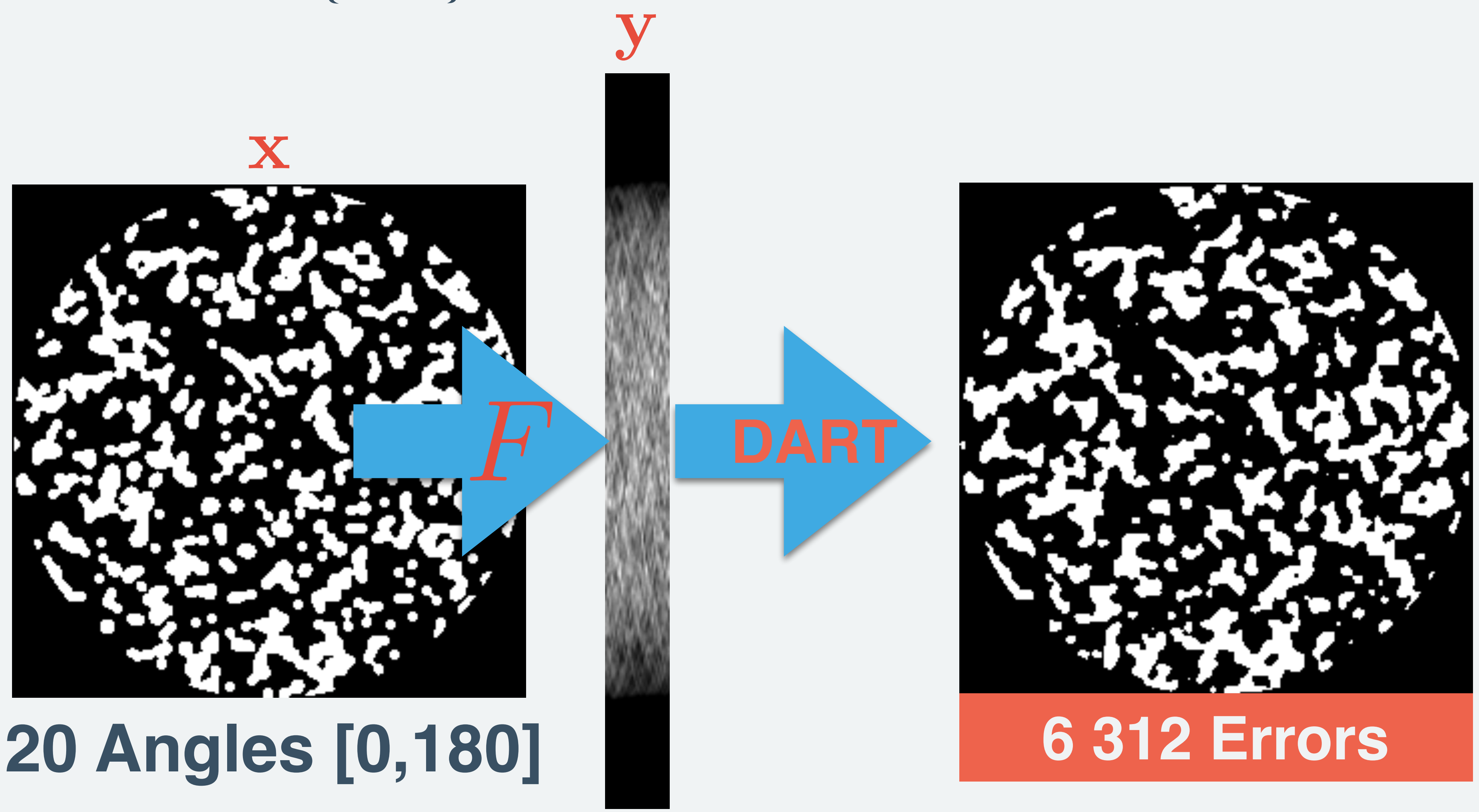
In electron tomography, the fidelity of the 3D reconstruction strongly depends on the employed reconstruction algorithm. In this paper, the properties of SIRT, TVM and DART reconstructions are studied with respect to having only a limited number of electrons available for imaging and applying different angular sampling schemes. A well-defined realistic model is generated, which consists of tubular domains within a matrix having slab-geometry. Subsequently, the electron tomography work-flow is simulated from calculated tilt-series over experimental effects to reconstruction. In comparison with the model, the fidelity of each reconstruction method is evaluated qualitatively and quantitatively based on global and local edge profiles and resolvable distance between particles. Results show that the performance of all reconstruction methods declines with the total electron dose. Overall, SIRT algorithm is the most stable method and insensitive to changes in angular sampling. TVM algorithm yields significantly sharper edges in the reconstruction, but the edge positions are strongly influenced by the tilt scheme and the tubular objects become thinned. The DART algorithm markedly suppresses the elongation artifacts along the beam direction and moreover segments the reconstruction which can be considered a significant advantage for quantification. Finally, no advantage of TVM and DART to deal better with fewer projections was observed.



Discrete ART (DART)

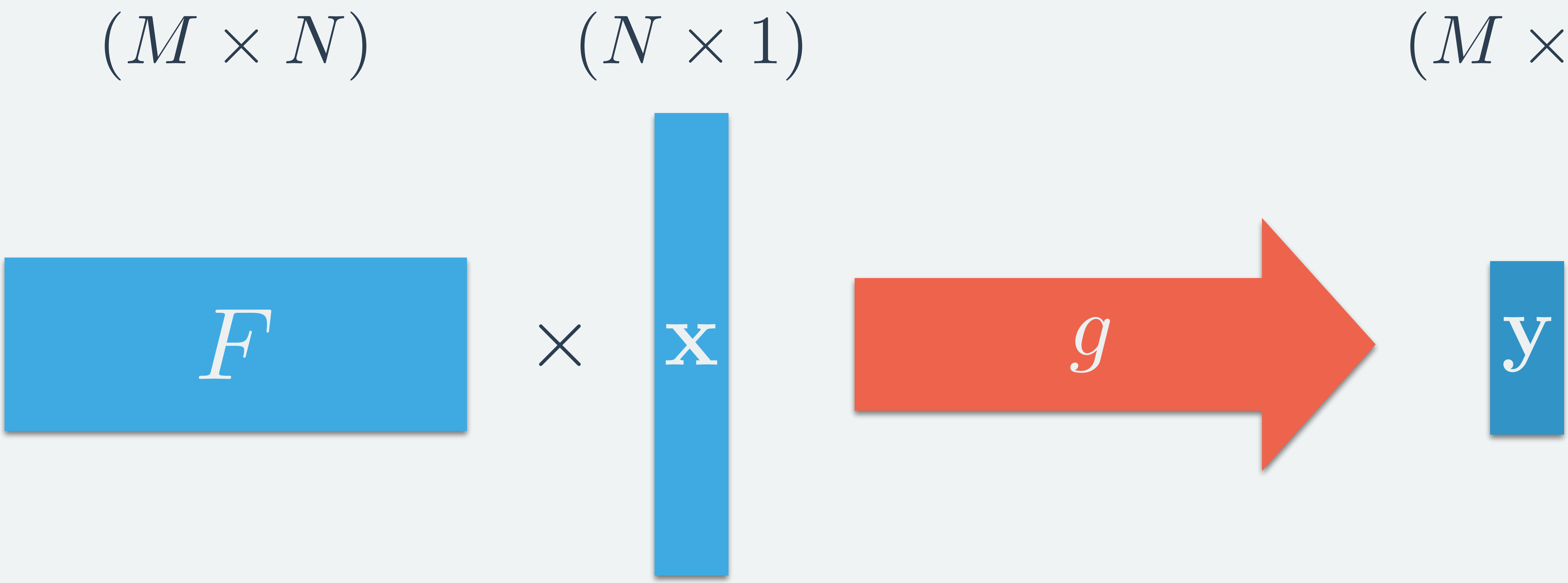
Prior Information

Elements (pixels/voxels) of the volume belong to a small set of values. Here: {0,1}.



Inverse Problems

General Linear Problem: $\mathbf{y} = g\left(F\mathbf{x}\right)$



Projection Matrix

- *iid* Random ?
- Underdetermined?
- Low Rank?
- Sparse?

Signal

Prior Model?

Channel

- Corruption
- Information Loss
- Noise Model?

Measurements

Observed Data

Ex: Compressed Sensing

$$\mathbf{y} = F\mathbf{x} + \mathbf{w} \qquad w_\mu \sim \mathcal{N}(0, \Delta)$$

CS Problem: How do we obtain **x** from **y** and **F** knowing **g = AWGN** & **x** is K-Sparse?

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \quad ||\mathbf{x}||_0 \quad \text{s.t.} \quad ||\mathbf{y} - F\mathbf{x}||_2^2 \leq \epsilon$$ (Greedy)

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \quad ||\mathbf{y} - F\mathbf{x}||_2^2 + \lambda ||\mathbf{x}||_1$$ (LASSO)

Deterministic

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \quad P(\mathbf{x}|\mathbf{y}, F)$$ (MAP)

$$\hat{\mathbf{x}} = \mathbb{E}[\mathbf{x}] = \int \mathrm{d}\mathbf{x} \;\; \mathbf{x} \, P(\mathbf{x}|\mathbf{y}, F)$$ (MMSE)

Probabilistic

Posterior Probability

Full Posterior

$$P(\mathbf{x}|\mathbf{y},F) = \frac{1}{Z}\prod_i P_0(x_i)\prod_\mu \frac{1}{\sqrt{2\pi\Delta}}\exp\left\{-\frac{1}{2\Delta}\left(y_\mu - \sum_i F_{\mu i}x_i\right)^2\right\}$$

Prior Model

AWGN Observation Model

Normalization *(intractable)*

$$Z = \int \mathrm{d}x_1 \int \mathrm{d}x_2 \ldots \int \mathrm{d}x_N \;\; \prod_i P_0(x_i)\prod_\mu \frac{1}{\sqrt{2\pi\Delta}}\exp\left\{-\frac{1}{2\Delta}\left(y_\mu - \sum_i F_{\mu i}x_i\right)^2\right\}$$

BP & Combining Priors

For Binary Images:
Can use an Ising prior and solve for MMSE solution using Belief Prop.

Key:
Prior Model is both **discrete** and enforces **smoothness.**

Belief Propagation Reconstruction for Discrete Tomography

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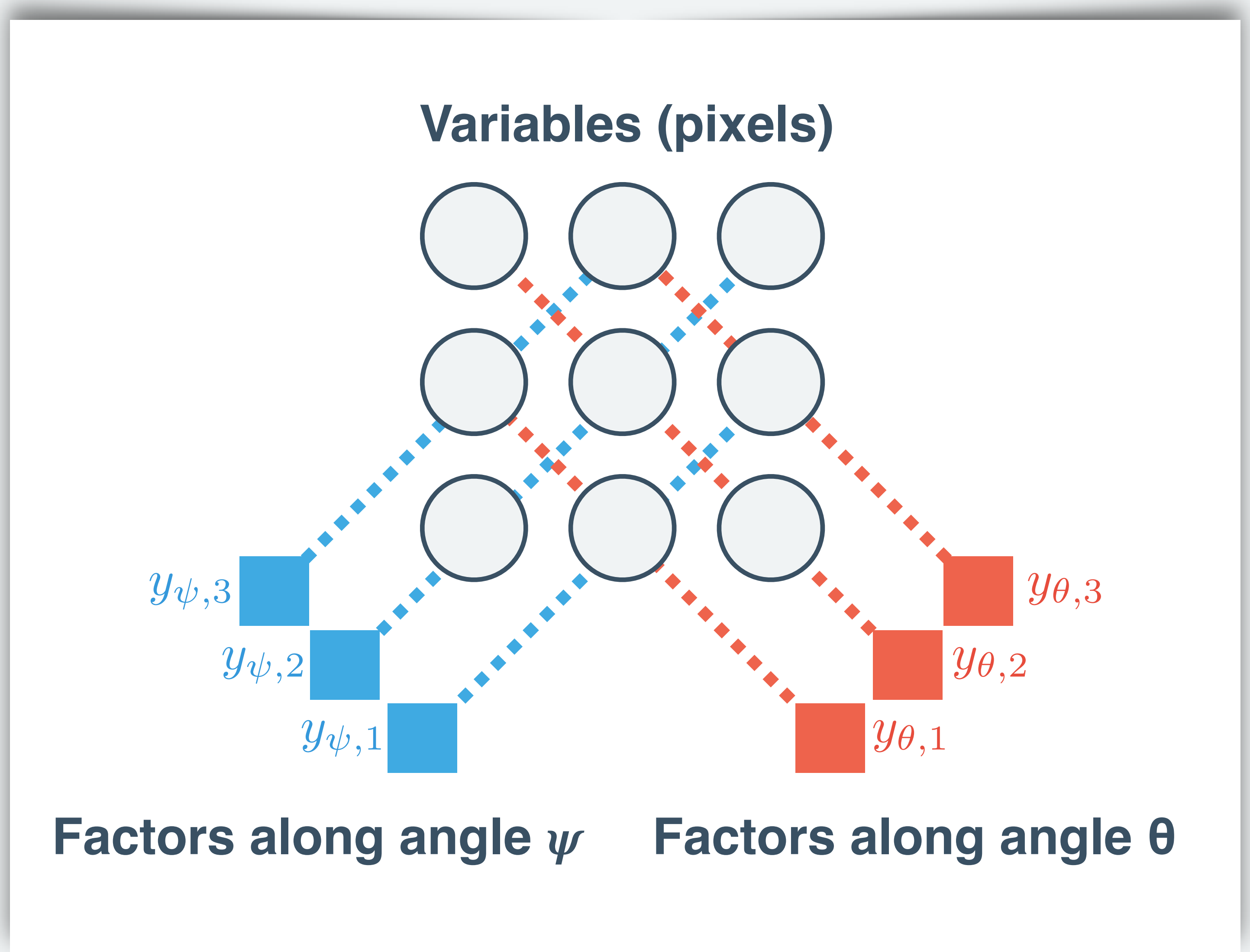
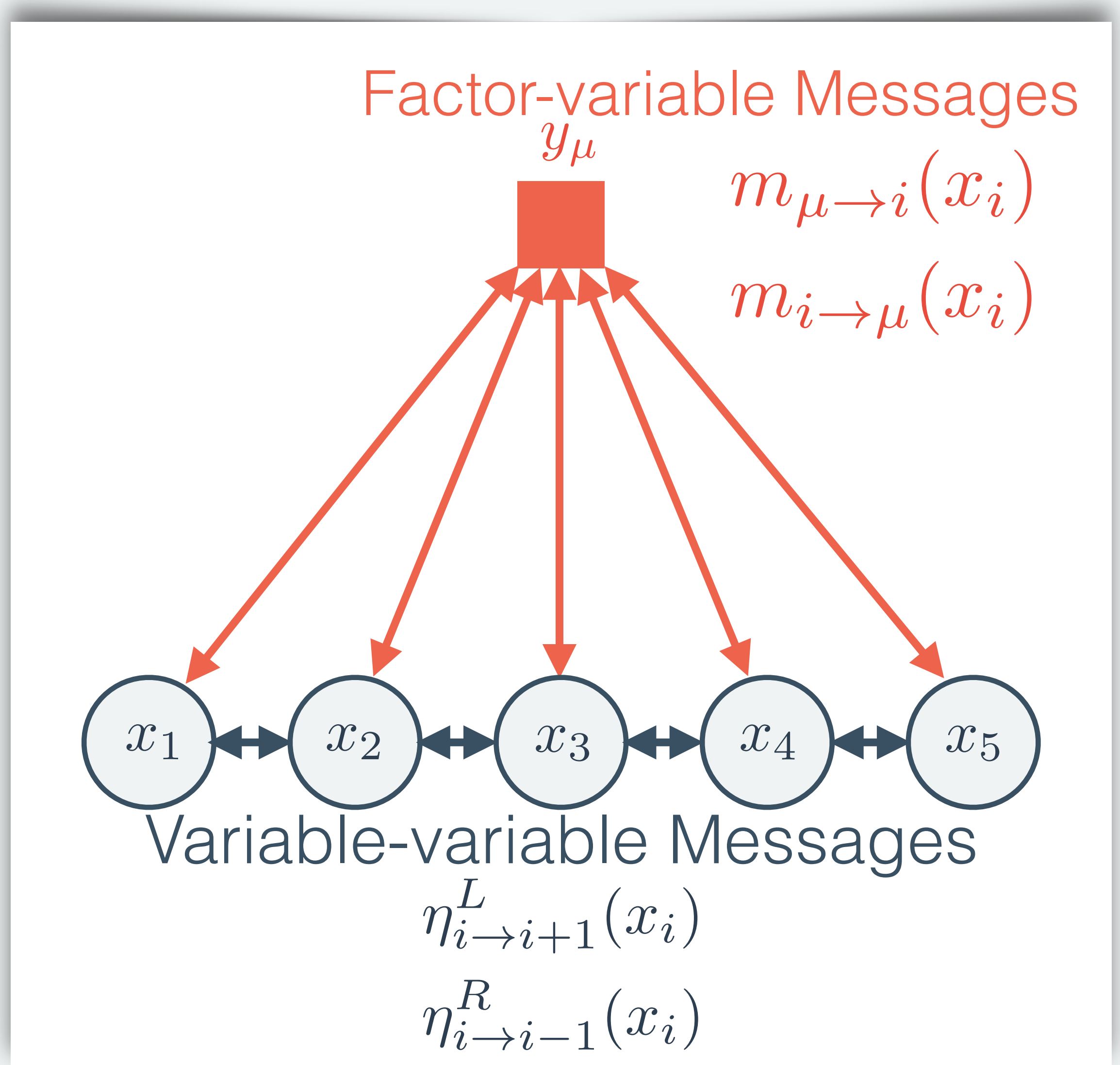
Abstract.
We consider the reconstruction of a two-dimensional discrete image from a set of tomographic measurements corresponding to the Radon projection. Assuming that the image has a structure where neighbouring pixels have a larger probability to take the same value, we follow a Bayesian approach and introduce a fast message-passing reconstruction algorithm based on belief propagation. For numerical results, we specialize to the case of binary tomography. We test the algorithm on binary synthetic images with different length scales and compare our results against a more usual convex optimization approach. We investigate the reconstruction error as a function of the number of tomographic measurements, corresponding to the number of projection angles. The belief propagation algorithm turns out to be more efficient than the convex-optimization algorithm, both in terms of recovery bounds for noise-free projections, and in terms of reconstruction quality when moderate Gaussian noise is added to the projections.

$$P(\mathbf{x}|\mathbf{y}) = \frac{1}{\mathcal{Z}} \prod_{\mu=1}^{M} \phi\left(y_\mu - \sum_{i\in\mu} x_i\right) e^{J_\mu \sum_{(i,j)\in\mu} \delta_{x_i,x_j}}$$

BP & Combining Priors

Each factor measures one **line** of pixels.



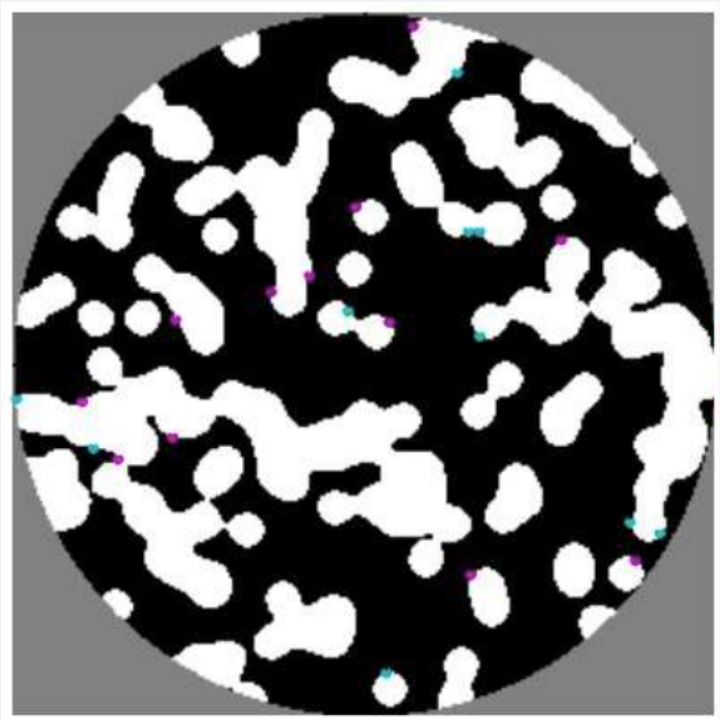


States along lines should be correlated with **neighbors.**

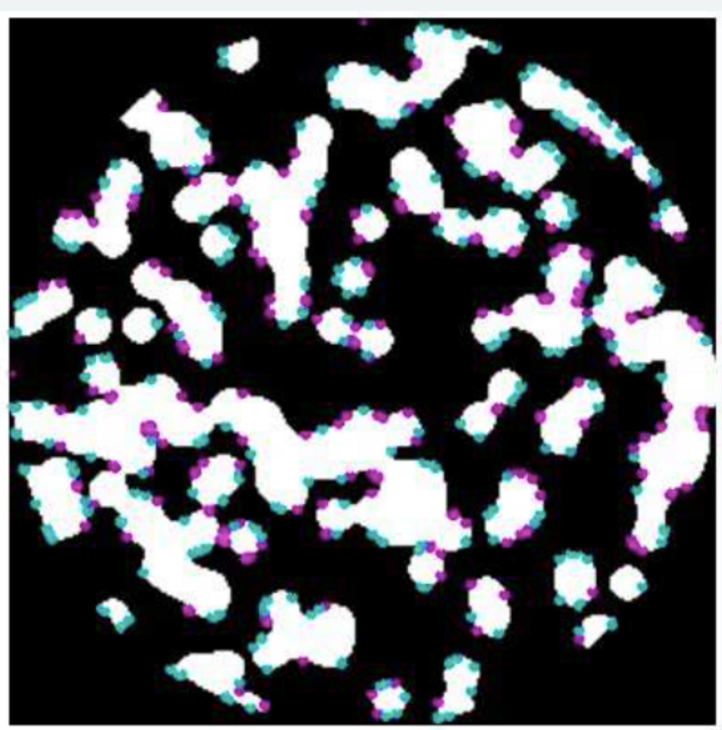
BP & Combining Priors

$\alpha = 1/10, \sigma/L = 0.006$

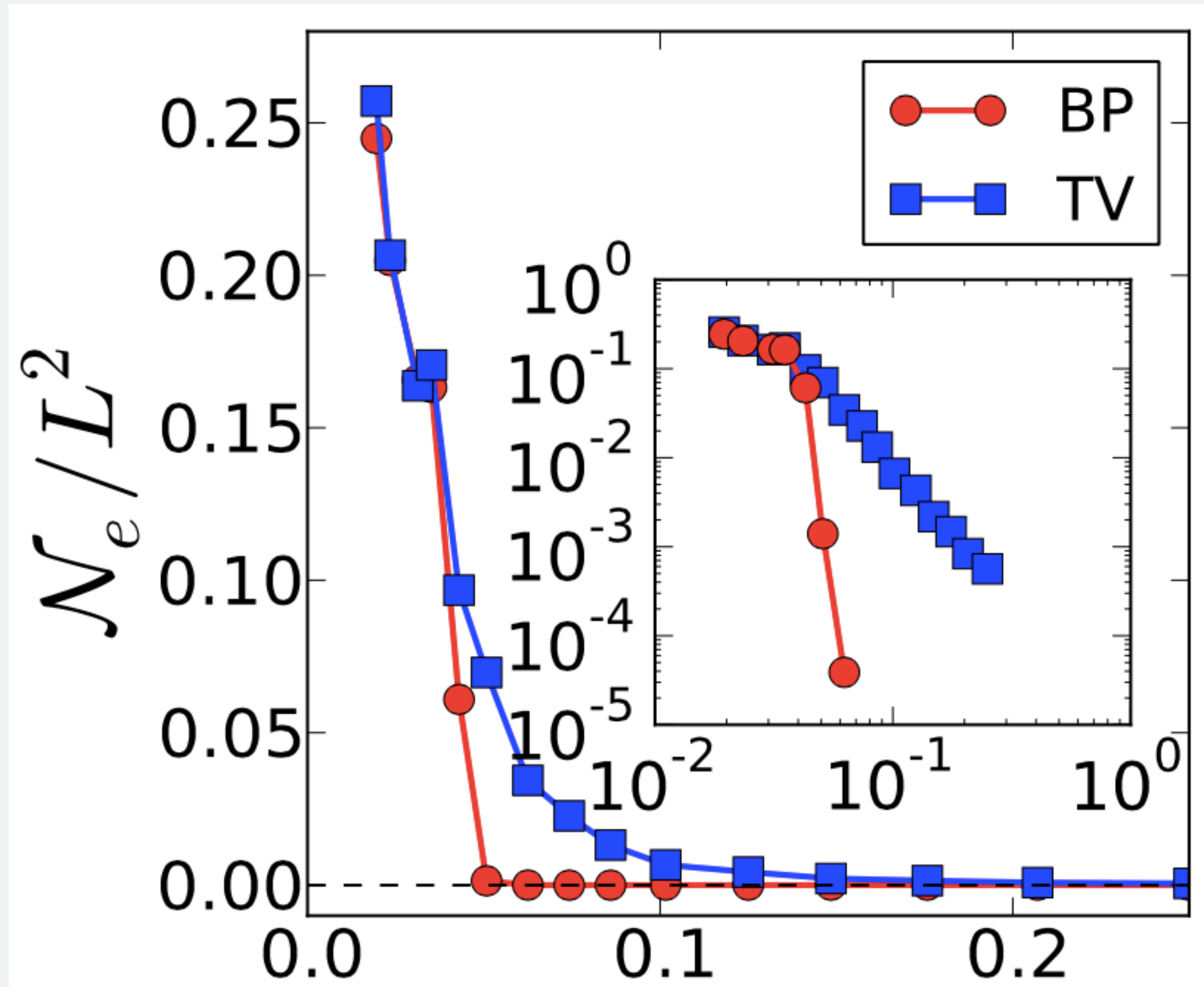
BP



TV



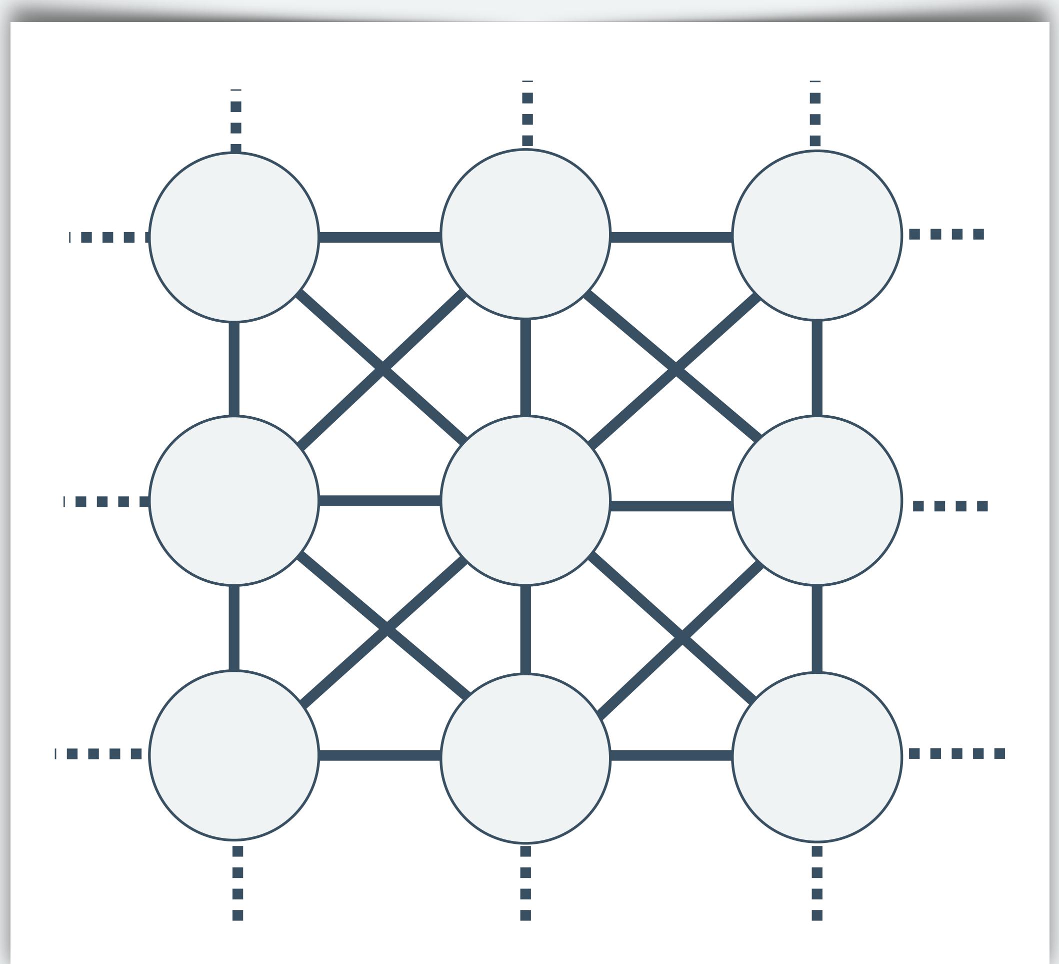
(Gouillart et al, 2013)



(Gouillart et al, 2013)

Modifying the Prior

Lattice Correlations. A full model of the entire signal that incorporates local correlations. *(related: MRFs)*



Caution Many tight loops, we cannot expect perfection.

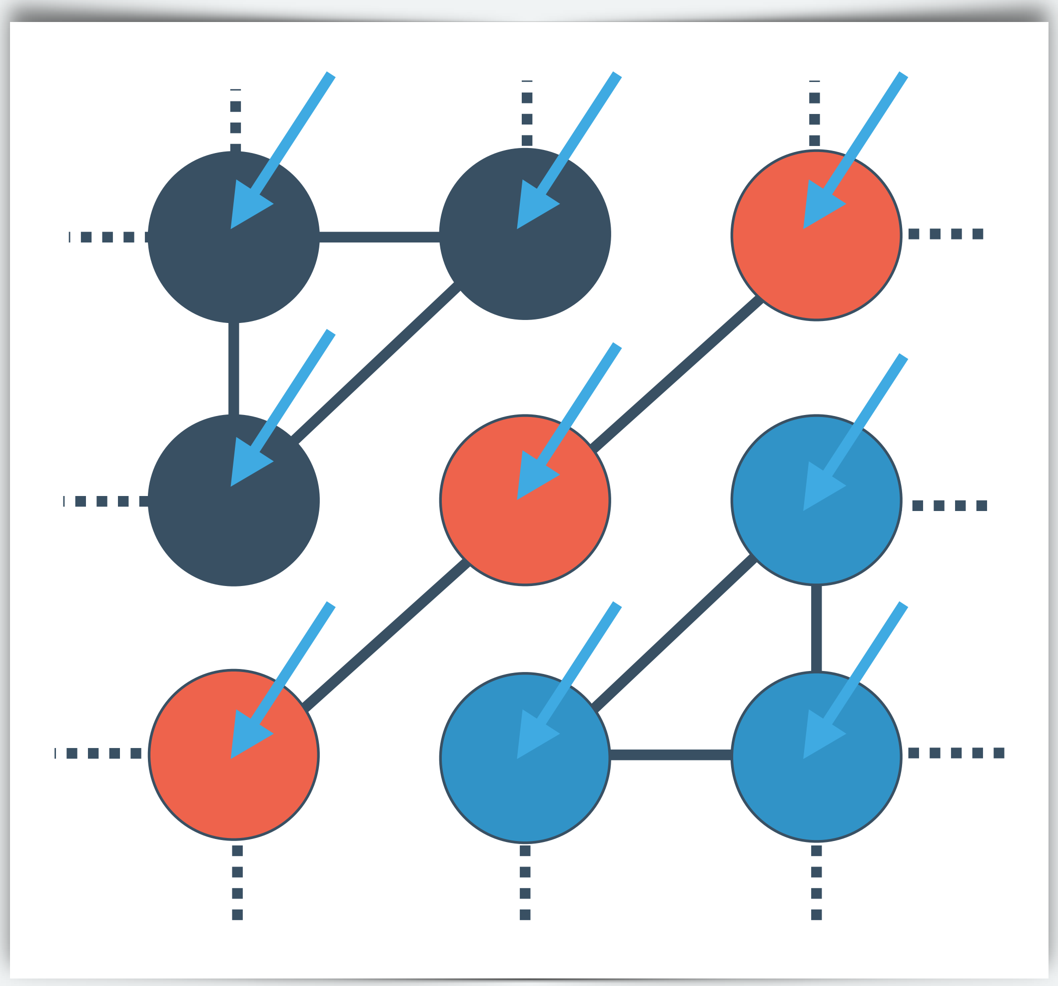
Advantages

- Perhaps a more accurate image model
- Adaptable correlation model (edges & weights) that can possibly be trained to exemplars
- Known results from familiar models
- Prior model is not dependent on sampling scheme.

Standard Potts Model Prior

A Potts Model We can generalize the Ising model as a two-state Potts model. The Potts model allows us to model any number of possible states (gray levels).

$$P_0(\mathbf{x}) = \frac{1}{\mathcal{Z}} e^{-\mathcal{H}(\mathbf{x})} \quad \text{for} \quad x_i \in \{\tau_1, \tau_2, \ldots, \tau_Q\}$$



Penalize Differing Neighbors

$$-\mathcal{H}(\mathbf{x}) = \eta \sum_{\langle i,j \rangle} \delta(x_i, x_j) + \sum_i h(x_i)$$

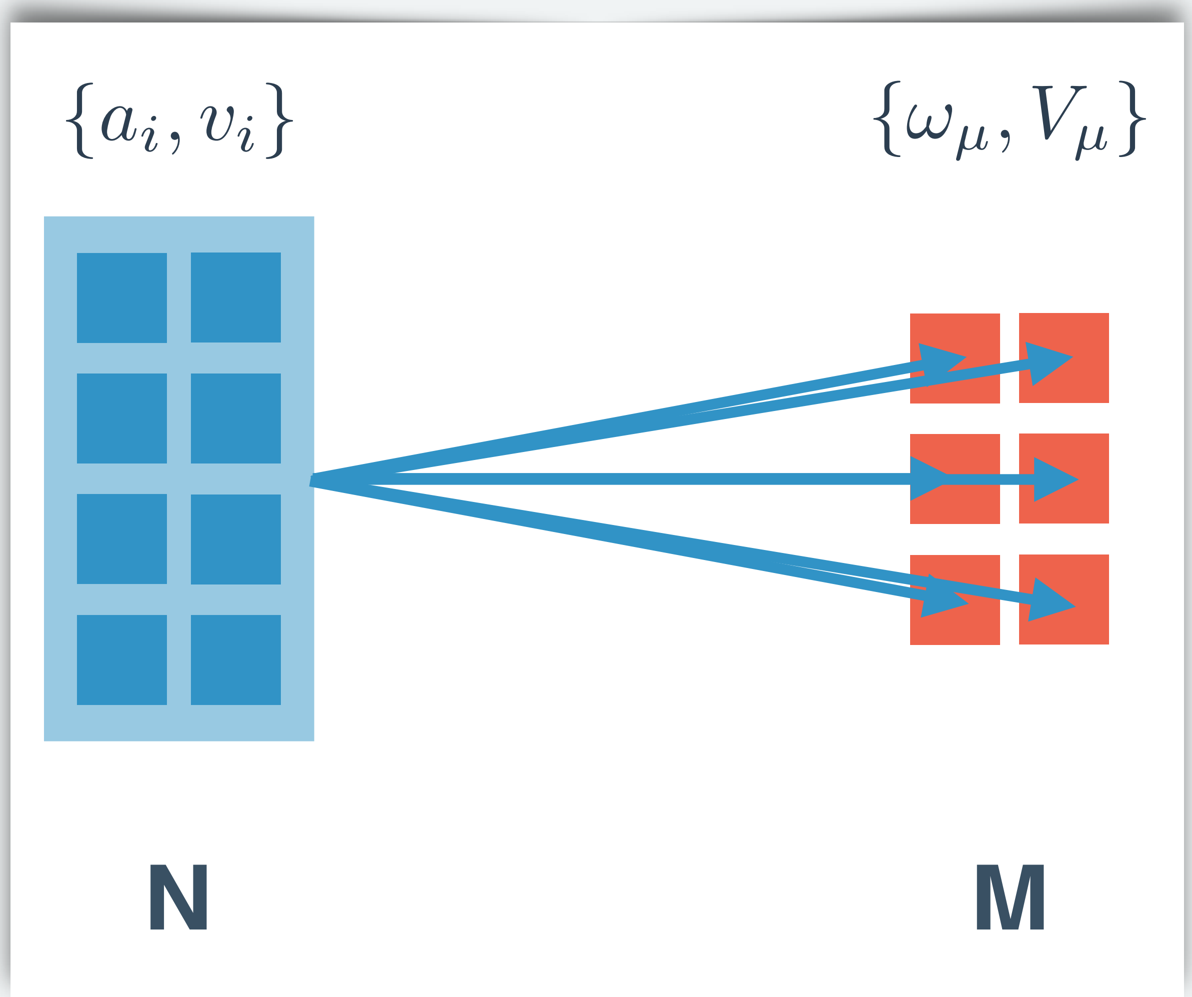
Some local biasing

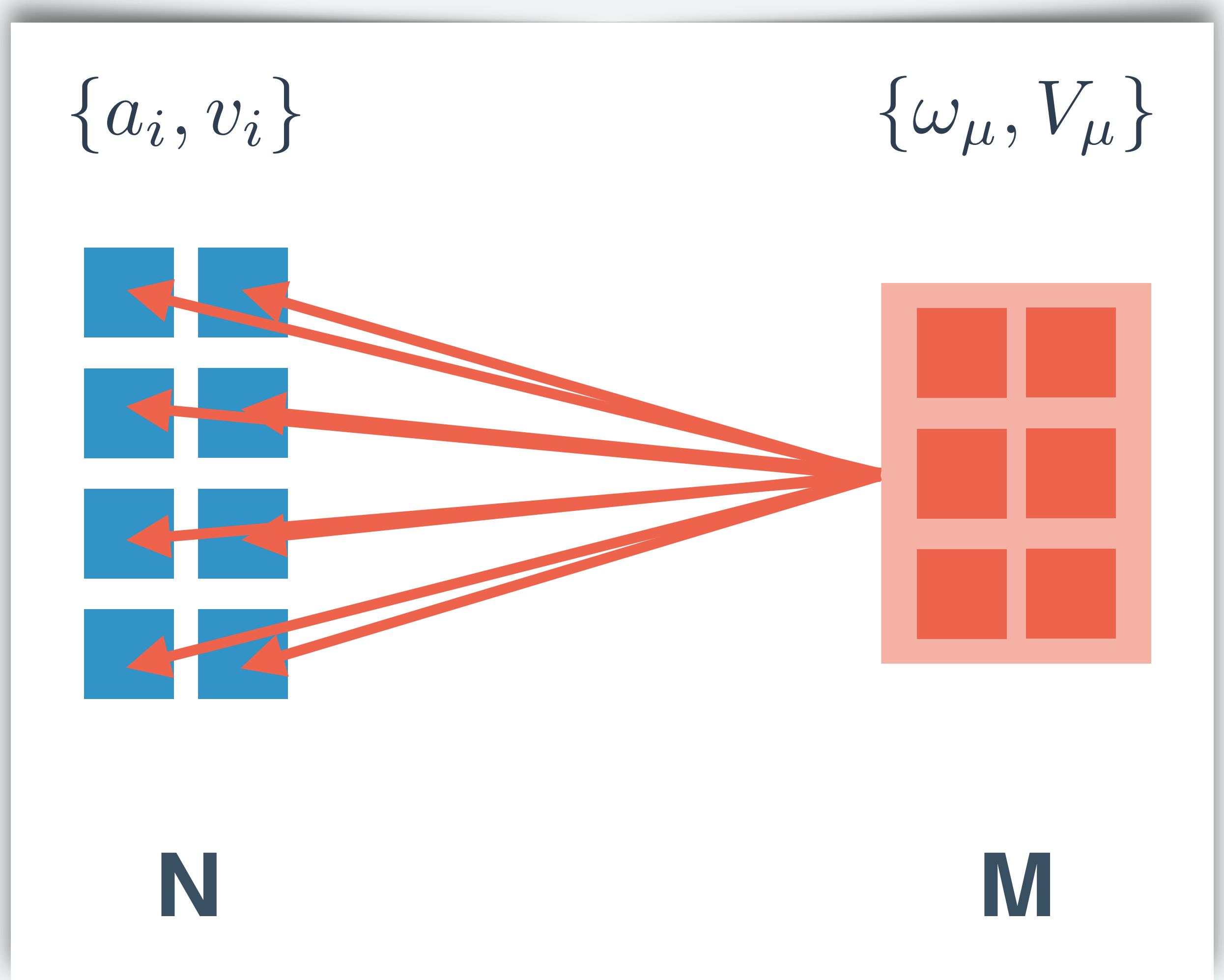
Direct Problem: Can solve Potts systems with Extended mean-field (Onsager Correction).

BP to AMP via TAP

TAP Intuition (Extended Mean-Field)

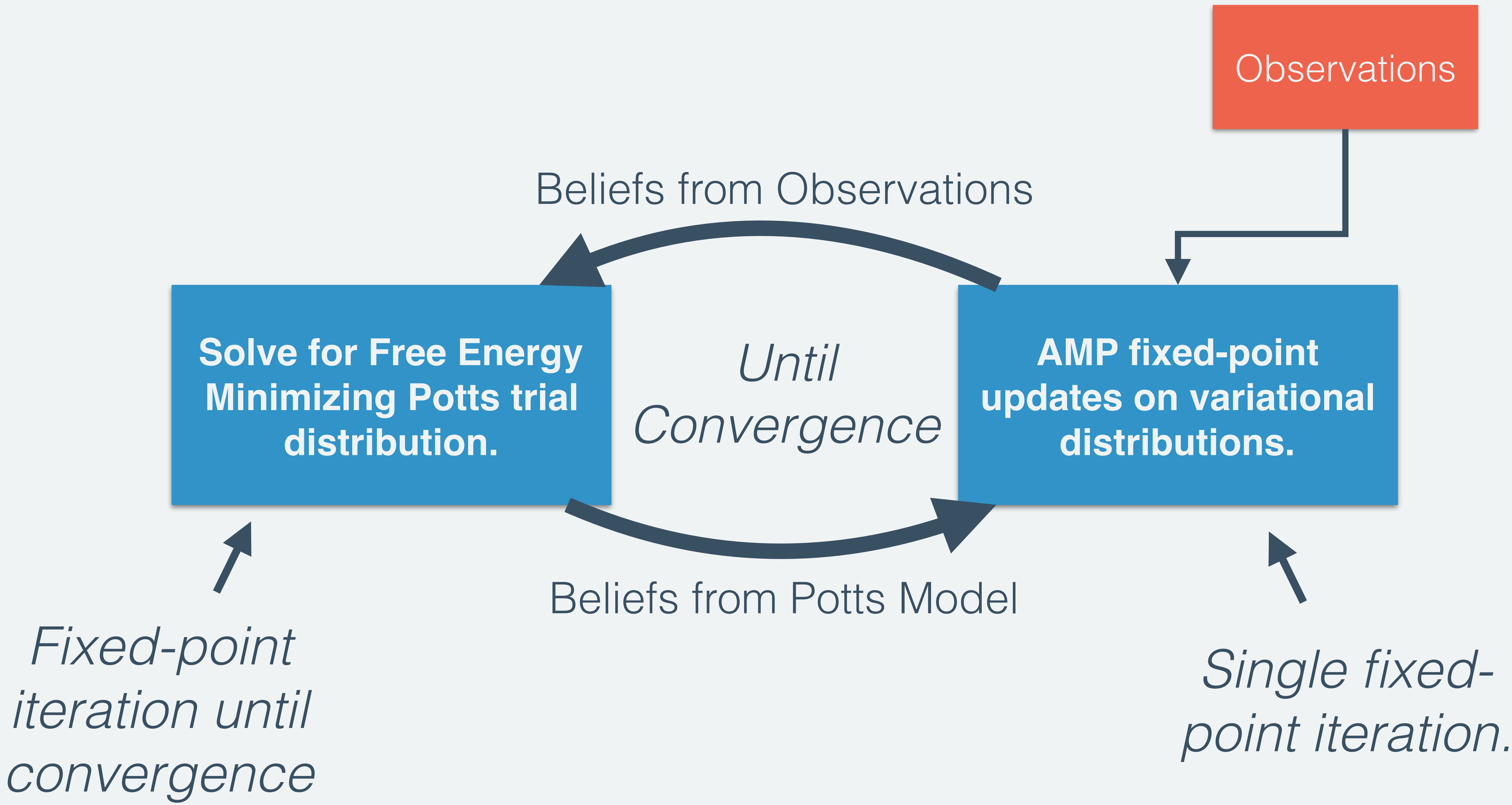
If **F** is ***not sparse*** and if its entries scale ***O(1/√N)***, then message means and variances are ***nearly independent*** of any single edge message in the limit ***N→∞***.





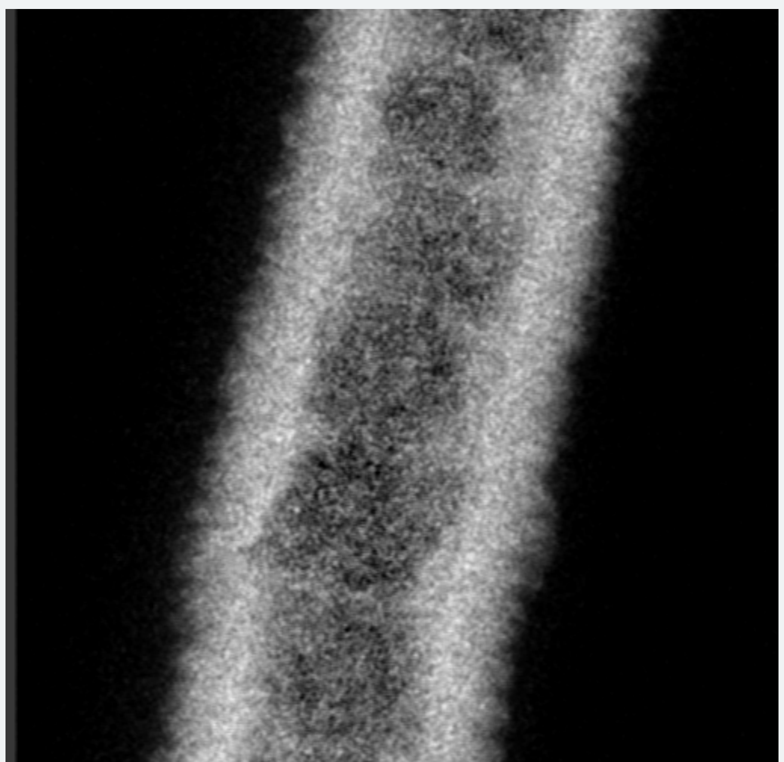
Big Savings: Compute Burden $O(\alpha N^2) \to O((1+\alpha)N)$

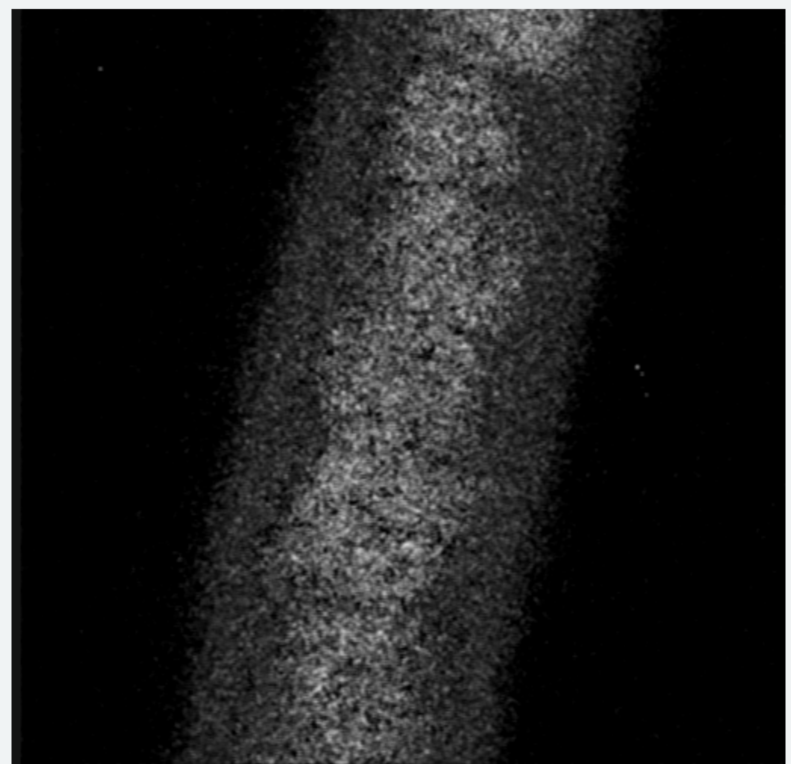
Potts+AMP

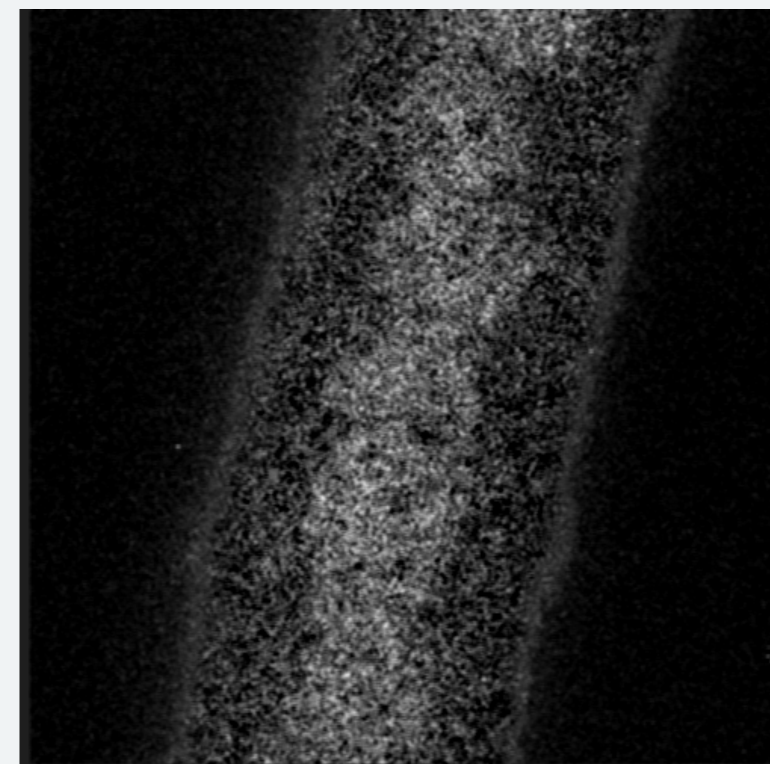

Observations
Beliefs from Observations
Solve for Free Energy Minimizing Potts trial distribution.
Until Convergence
AMP fixed-point updates on variational distributions.
Beliefs from Potts Model
Fixed-point iteration until convergence
Single fixed-point iteration.

Preliminary Results: Dataset

Carbon Nanotube containing **CoO** crystals
HAADF-STEM in Chemical Mode *(low SNR from binning)*
49 viewing angles between +\- 62.52deg
512x512 resolution micrographs *(downsampled to 129x129)*


Carbon

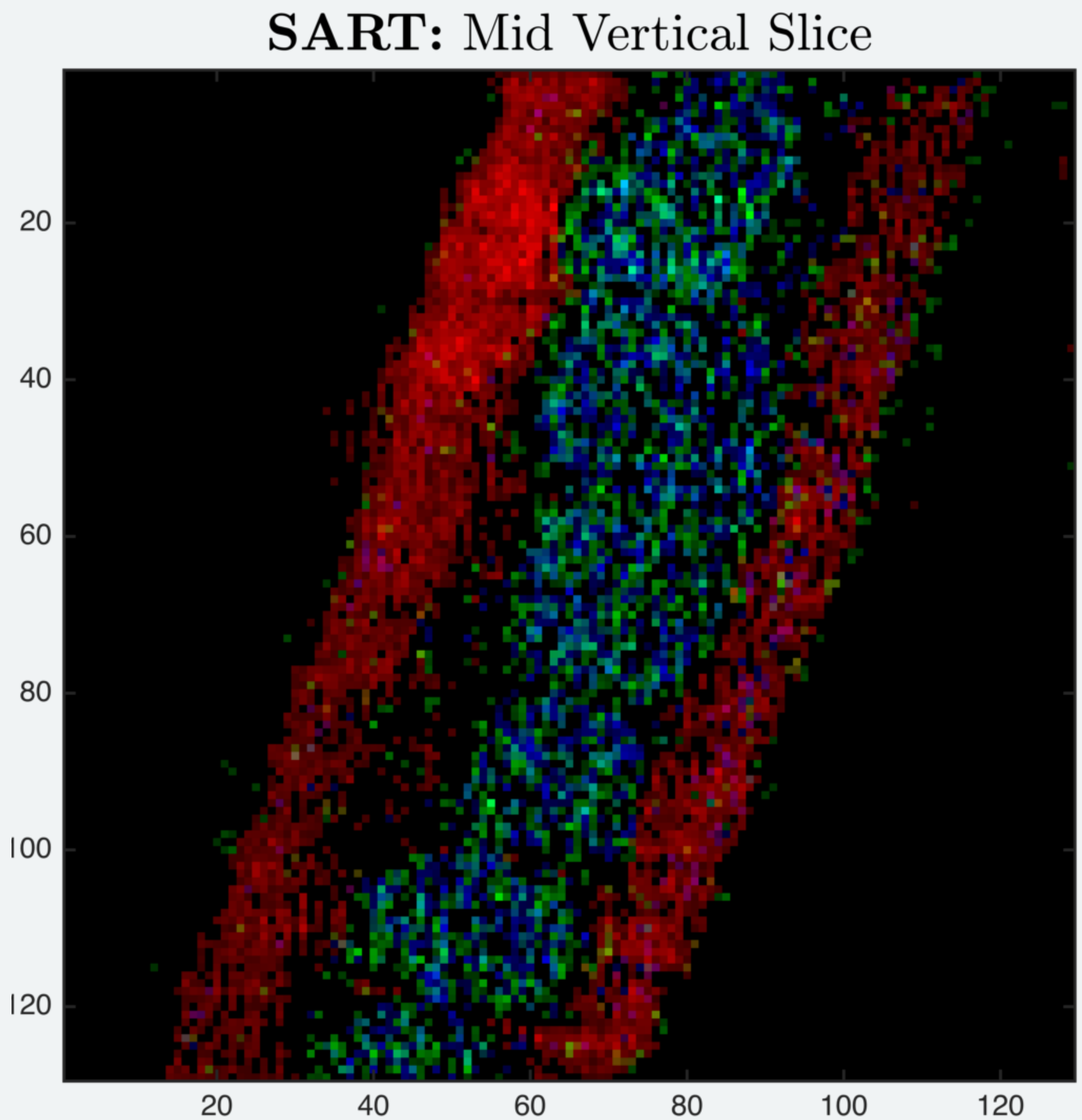

Cobalt

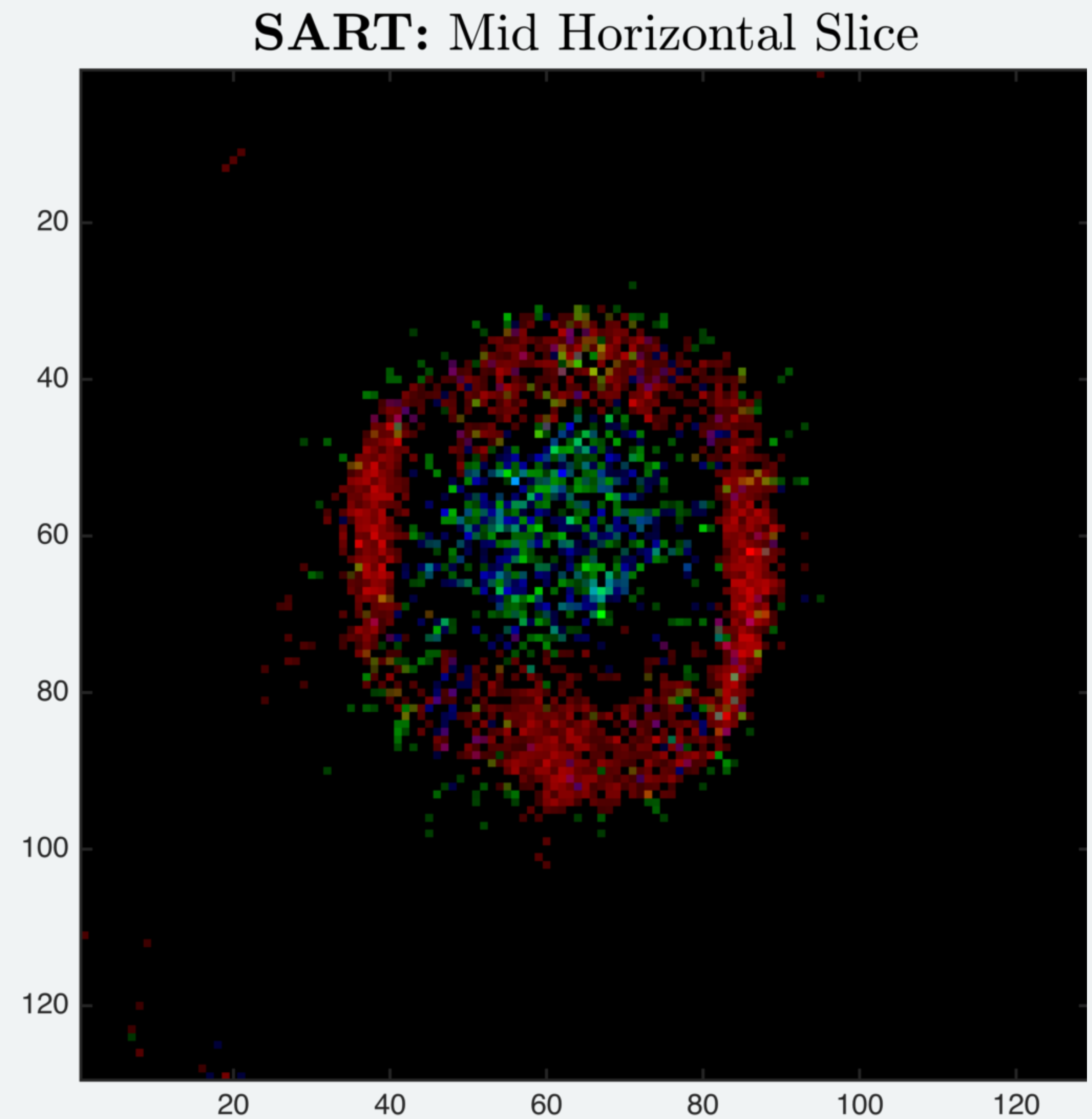

Oxygen

Data: *Acquired @ IPCMS, Université de Strasbourg*

Preliminary Results: Composite

Recovered Volume: 129x129x129



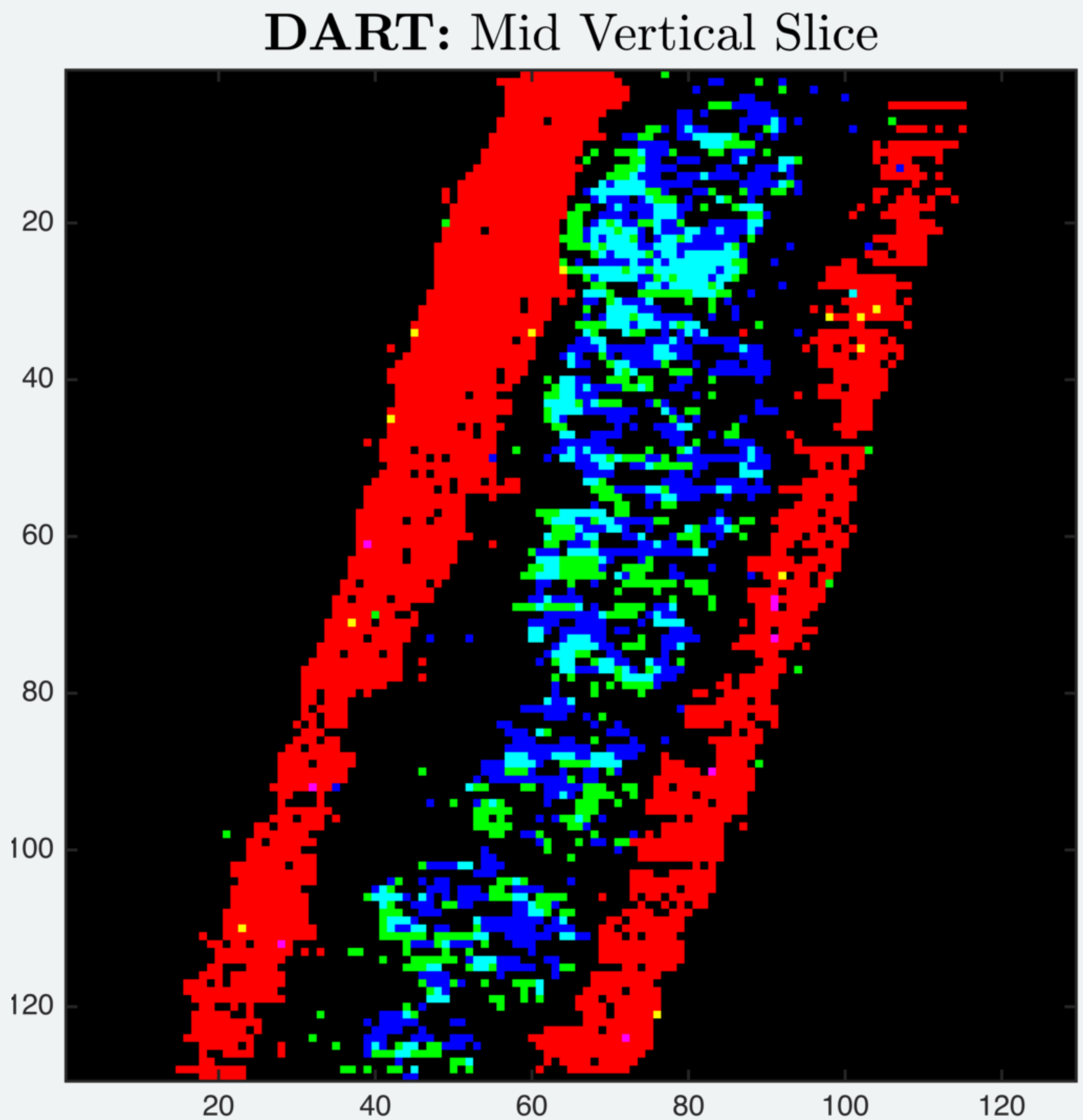


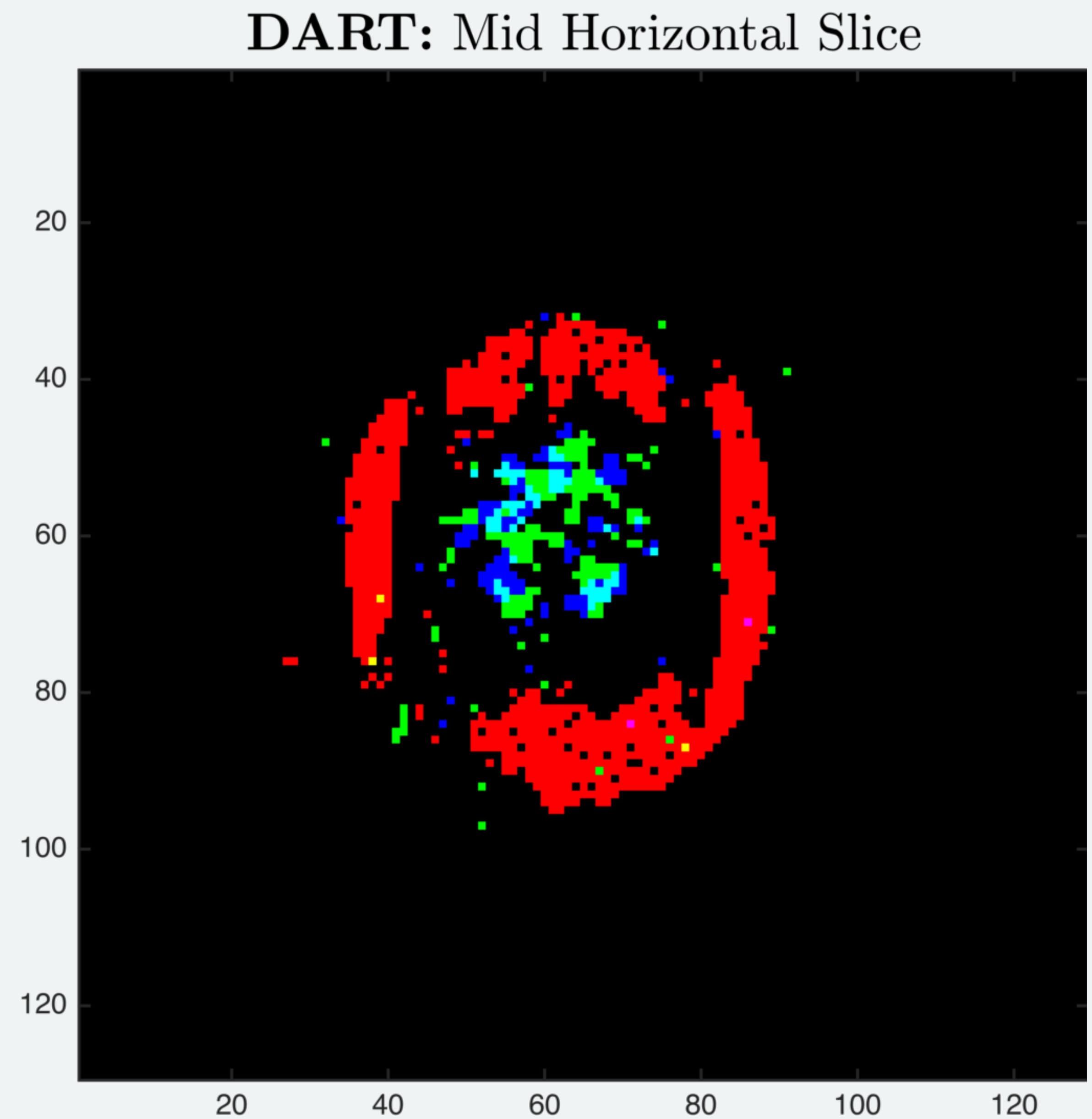
25 Iterations.
Random projection update order.

Carbon | Cobalt | Oxygen

Preliminary Results: Composite

Recovered Volume: 129x129x129

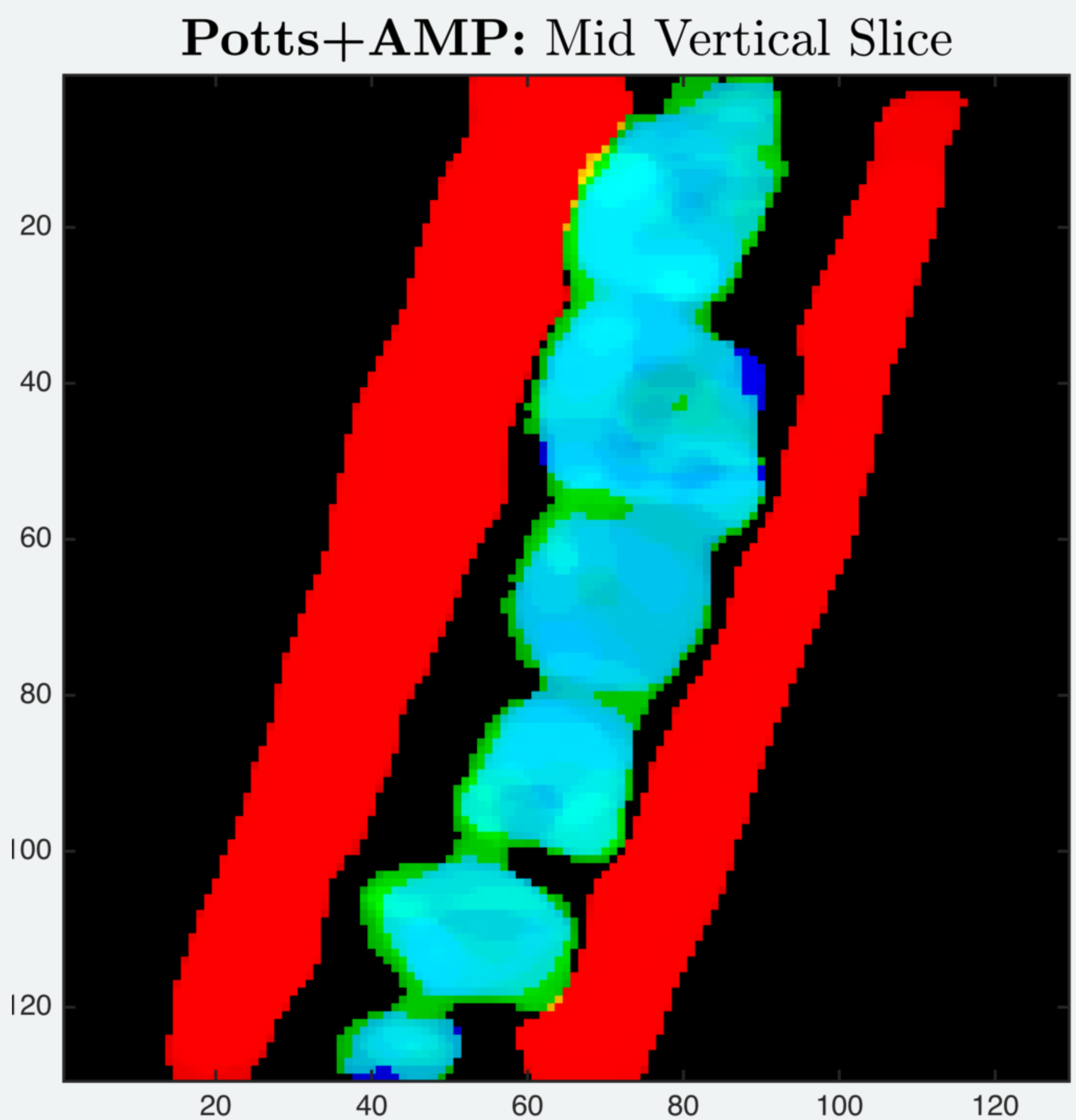




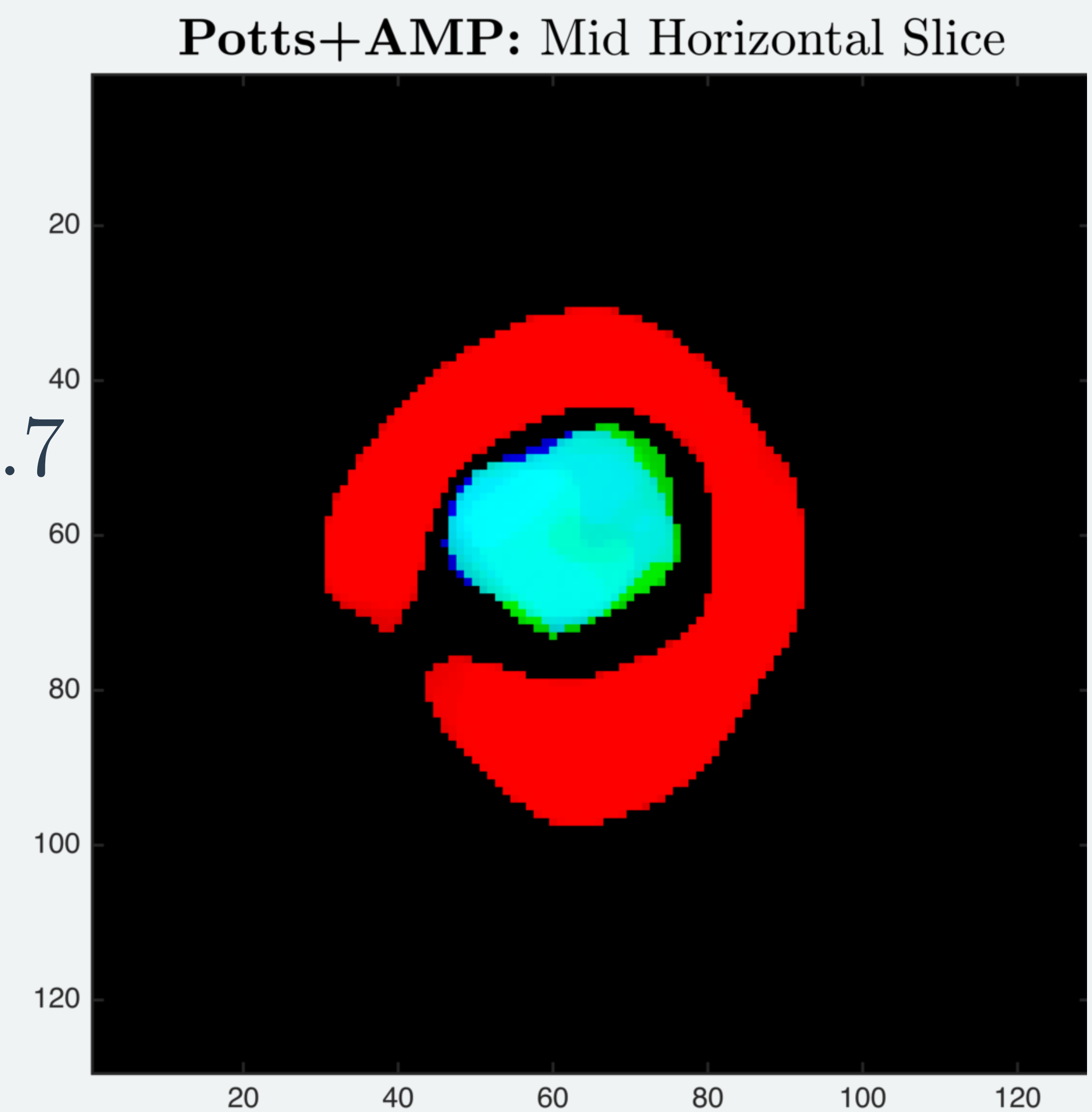
Initialized with 25 iteration SART recovery.
4 Colors per element recovery.
Interior ARM: 25 iteration SART.
“Unfix” probability: 0.95
10 DART iterations (converged quickly)

Preliminary Results: Composite

Recovered Volume: 129x129x129



$$\eta = 0.7$$

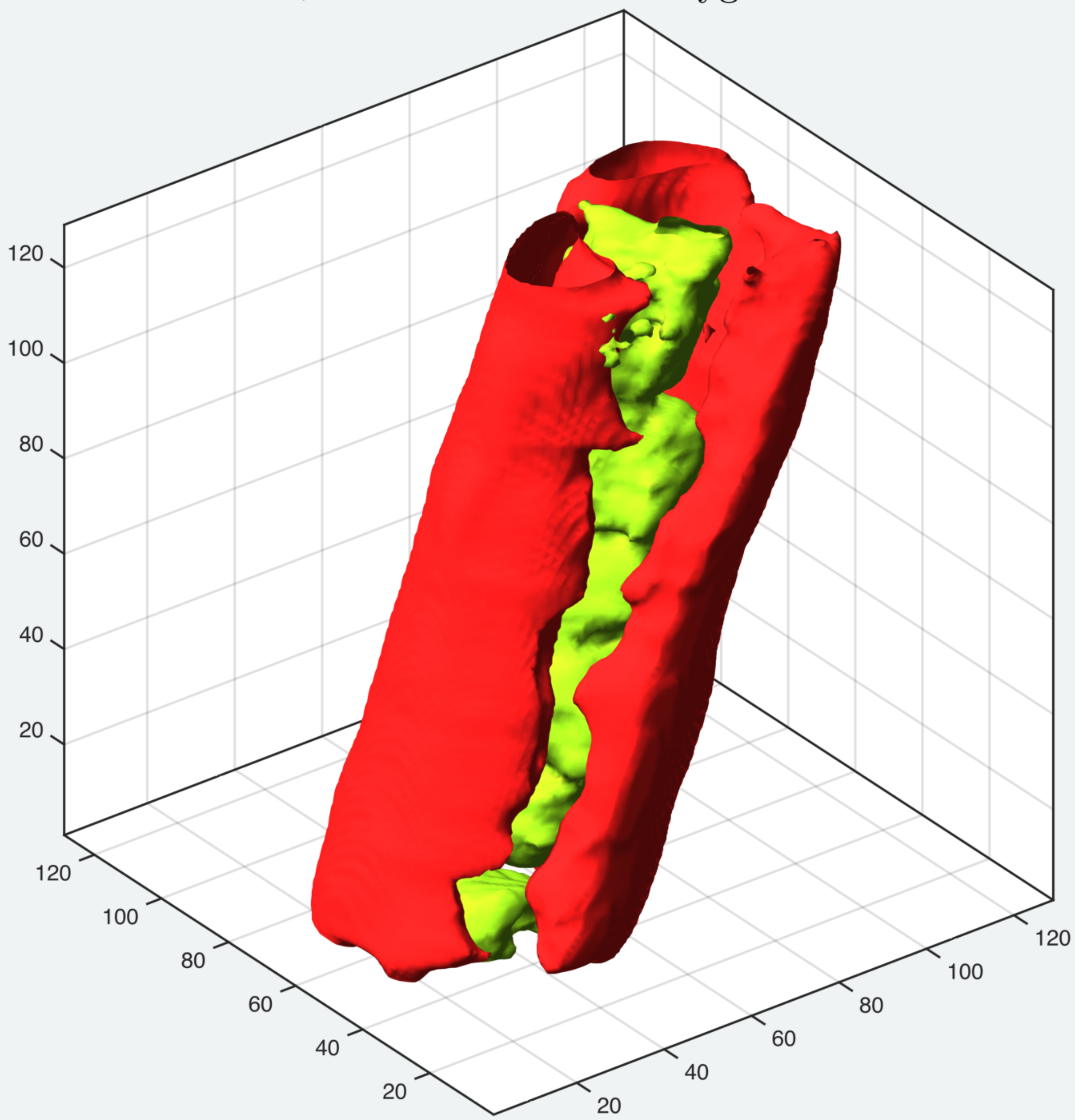


30 AMP Iterations
20 inner Potts/TAP iterations
4 Colors per element recovery
Noise Variance learned online

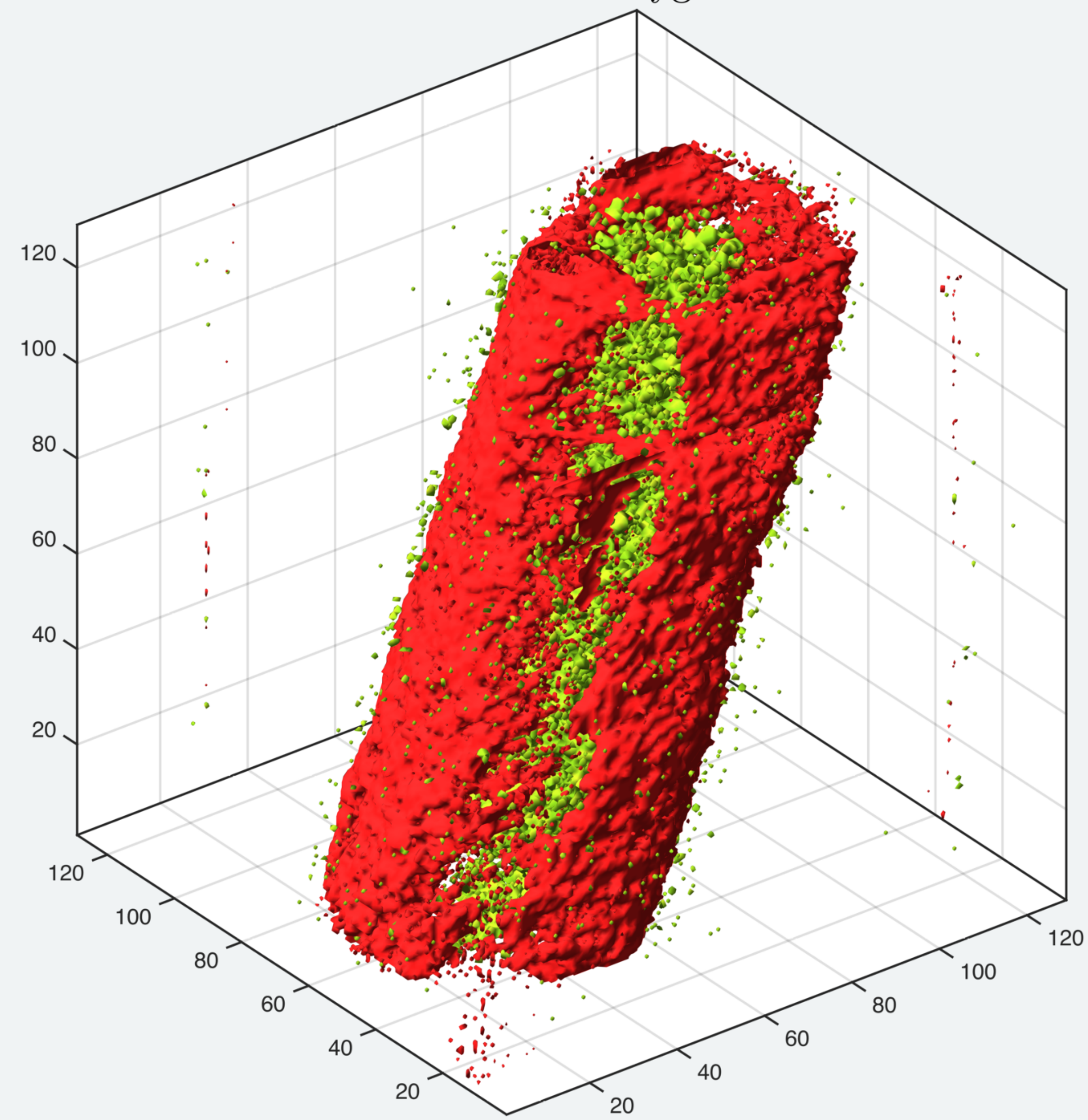
Carbon | Cobalt | Oxygen

Preliminary Results: Composite

Potts+AMP: Carbon & Oxygen Volumes



DART: Carbon & Oxygen Volumes



Conclusions

Potts+AMP

- ✓ Can be used in more general settings, i.e. different noise channels.
- ✓ Adaptable lattice structure.
- ✓ Incorporates both discrete and structured priors.
- ✓ Extensible to hierarchical prior models.
- ➡ Still many free parameters to tune (coupling strength, etc.)
- ➡ Efficiency still a hindrance.

Open Questions

- ◉ Can the coupling be learned on-line?
- ◉ Can the alphabet size and values be learned *a posteriori* ?
- ◉ Can adaptive damping aid convergence speed?
- ◉ What is the best noise model for HAADF-STEM?

SPHINX @ENS

Statistical **PH**ysics of **IN**formation e**X**traction

«OU»

Statistical **PH**ysics of **IN**verse comple**X** sysems

ENS

1794

Questions?

Merci!

Collaborators

Florent KRZAKALA — LPS-ENS

Ovidiu ERSEN — IPCMS-Uni.Stra.

Simona MOLDOVAN — IPCMS-Uni.Stra.